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A qualitative study of problematic reasonings of undergraduate electrical engineering students in Continuous Time Signals and Systems courses

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A QUALITATIVE STUDY OF PROBLEMATIC REASONINGS OF UNDERGRADUATE ELECTRICAL ENGINEERING STUDENTS IN CONTINUOUS TIME SIGNALS AND SYSTEMS COURSES

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Farrah Fayyaz

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of

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To God who has given me every blessing and every ability

To my advisor for her endless support and guidance

To my father for believing in me

To my mother for our relationship which gave me strength to go through this process

To Tamania for her endless support

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ABSTRACT

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Continuous Time Signals and Systems is a core course in the undergraduate electrical engineering curriculum. The topics covered in this course are difficult to learn conceptually because a significant number of topics are abstract, disconnected from a student's daily life, and make extensive use of mathematical modeling and formulas. Engineering educators have put in significant effort to design effective educational strategies for this course; however, there remained a gap in qualitative understanding of students' reasonings associated with difficulties in conceptual learning of the course content. This study aims to fill this gap by identifying problematic reasonings of undergraduate electrical engineering students when they engage with the course content. In addition, this study aims to identify and assess the differences in the problematic reasonings employed by students of different academic statuses (students who have only taken Continuous Time Signals and Systems course and students who have taken subsequent courses). Looking at the differences in the problematic reasonings used by the students of different academic statuses enables an understanding of the persistent difficulties in learning the course content.

This study used a constructivist framework and started with the design and validation of a sixty-minute semi-structured interview protocol. The protocol is designed based on the difficult topics in this course identified through literature and content experts of Continuous Time Signals and Systems courses. Once the protocol was tested, nineteen undergraduate electrical engineering students from a teaching-intensive Midwestern university were interviewed for this study. The participants were required to have passed this course already. Of the nineteen participants, eight (CTSS-only group) have only taken Continuous Time Signals and Systems course and eleven (CTSS-plus group) have taken subsequent (up to 4) courses. Each student was interviewed individually. Data collected from think-aloud interviews were analyzed using thematic analysis.

Results revealed that the reasonings used by the participants that are potentially problematic in conceptual learning of Continuous Time Signals and Systems course content are related to content areas of Signal Representations and Operations, Frequency Analysis, and System Analysis. The identified reasonings can be further classified under three main learning challenges, namely i) accommodation; ii) translation of a signal to its multiple representations in one domain; and iii) translation of a signal to its multiple representations between domains (time and frequency). The robust problematic reasonings are the ones exhibited equally by all nineteen students in translating a signal to its multiple representations between domains (time and frequency).

The results of this study can provide a broader impact on future work across many subfields within engineering including electrical, computer, mechanical, biomedical, aeronautics, and astronautics. This study will benefit both engineering curriculum developers to design curriculum that efficiently help students develop a conceptual understanding of courses like Continuous Time Signals and Systems and instructors of Continuous Time Signals and Systems courses to develop successful educational strategies for this course. Additionally, the design of this study can be used as an example for future work in understanding problematic concepts within engineering education.

CHAPTER 1 - INTRODUCTION

Various factors influence student learning in educational settings. These include faculty (e.g., adequacy in professional knowledge, teaching style, attitude, sympathy, language skill, etc.), students (e.g., ability, attitude, need, learning styles, working memory capacity, motivational styles, etc.), physical situations, assessment methods, socio-cultural factors, and misconceptions (Bahar, 2003). Misconceptions are defined as "any aspect of an individual's conceptual understanding that resists conceptual change and contributes to an incorrect, naive, or unproductive conceptual understanding" (Streveler, Brown, Herman, & Montfort, 2014, p.1). Reasonings are the knowledge that students reveal (speak or write) in response to a particular problem which might just be limited within the context of the problem. Misconceptions can come from informal or formal educational settings and at any stage of the students' education (Bahar, 2003). In the literature, misconceptions are also referred to as naive beliefs (Caramazza, McCloskey, & Green, 1981), erroneous ideas (Fisher, 1985), preconceptions (Hashweh, 1988), multiple private versions of science (McClelland, 1984), underlying sources of error (Fisher & Lipson, 1986), personal models of reality (Klopfer, Champagne, & Gunstone, 1983), spontaneous reasoning (Viennot, 1979), persistent pitfalls (Meyer, 1987), common sense concepts (Halloun & Hestenes, 1985), spontaneous knowledge (Pines & West, 1986), alternative frameworks (Driver & Easley, 1978), faulty extensions of productive prior knowledge (Smith, diSessa, & Roschelle, 1994), and children's science (Gilbert, Osborne, & Fensham, 1982).

I am situating this study in a constructivist framework that is based on two assumptions. One, the reasonings employed by the students when they engage with a certain concept play role in the learning of that concept, and two, the knowledge that a student possesses may be "in-pieces" (DiSessa, 1983, 1988, 2008), which means a particular reasoning used by a student may not be representative of the complete mental model of that student. Based on these two assumptions, I choose the term problematic reasoning over misconception for this study and define problematic reasoning as a person's reasoning (purposeful effort to generate justifiable conclusions and make sense of the problem) that has the potential to hinder conceptual understanding and cultivate misconceptions.

Learning conceptual knowledge in engineering science is crucial to develop competence and expertise in engineering. An essential question within the domain of conceptual knowledge and learning is what makes some concepts difficult to learn (Perkins, 2007; Streveler, Litzinger, Miller, & Steif, 2008). So far, this question is not deeply explored in engineering education research (Streveler, Litzinger, Miller, & Steif, 2008). To date, the research in conceptual knowledge within engineering sciences have been focused mainly on force, heat, and electric current (Streveler, Litzinger, Miller, & Steif, 2008). A few key reasons that these concepts are difficult to learn are that they are abstract, not observable directly, and are usually miscategorized by novice learners (Chi, 2005). These difficulties are also inherent in most of the Continuous Time Signals and Systems course content.

Continuous Time Signals and Systems is a core course in the undergraduate electrical and computer engineering curriculum (Nasr, Hall, & Garik, 2005; Wage & Buck, 2001). The course content is difficult to learn conceptually because a significant number of topics in this course are abstract, disconnected from a student's daily life, and make extensive use of mathematical modeling and formulas (Nasr, Hall, & Garik, 2005; Ferri et al., 2009; Han, Zhang, & Qin, 2011; Tsakalis et al., 2011). The conceptual understanding of the content of this course is important as these concepts become foundational knowledge for many other courses in undergraduate electrical and computer engineering curriculum like communication, control systems, circuit design, image, and audio processing (Oppenheim, Willsky, & Nawab, 1997).

Electrical engineering educators have put in significant effort to design effective educational strategies for this course. These include the design of the Signals and Systems Concept Inventory (SSCI) (Wage & Buck, 2001), use of SSCI to study difficult concepts in Signals and Systems (Wage, Buck, & Wright, 2004), use of computer tools and simulations (Cavicchi, 2005; Han, Zhang, & Qin, 2011), and other active learning strategies (Ferri et al., 2009). Additionally, there had been a few quantitative studies to assess students' performance in Signals and Systems (Huettel, 2006; Ogunfunmi, 2011). Despite all these efforts, the course content has continued to challenge students' learning as evidenced by well-above-average drop/failure rates (Simoni, Aburdene, & Fayyaz, 2013a, 2014). This is because there is a lack of exploratory studies on students' understanding of this course (Nasr, 2007).

Teaching for conceptual change and development of successful learning environments and pedagogical strategies necessitates exploration of students' misconceptions and concepts that are difficult to learn (Streveler, Olds, Miller, & Nelson, 2003; Anderson, Abell, & Lederman, 2007; Streveler, Brown, Herman, & Montfort, 2014). Open-ended and theoretically-focused conceptual change research in engineering education is most suitable for investigation of existence, importance, and interrelatedness of a cognitive phenomenon (Streveler, Brown, Herman, & Montfort, 2014). To develop most appropriate educational strategies for Signals and Systems there is a need for exploratory studies on how students engage with Continuous Time Signals and Systems course content (Nasr, Hall, & Garik, 2007). The goal of this study is to fill the gap of qualitative analysis of students' conceptual understanding of Signals and Systems.

1.1 Purpose

The purpose of this qualitative study is to understand the problematic reasonings used by undergraduate electrical engineering students when they attempt to access Continuous Time Signals and Systems course content. Additionally, this study seeks to understand how the problematic reasonings change when students progress in their academic career in engineering. This will help to understand students' unintended approaches in Continuous Time Signals and Systems course content that persist to challenge learning even after students' continued use of these concepts in more contexts and applications. The research questions guiding this study will be presented next.

1.2 Research Questions

This study has the following research questions:

RQ 1. What problematic reasonings do undergraduate electrical engineering students employ when they engage with the Continuous Time Signals and Systems course content?

RQ 2. How do these problematic reasonings differ after the students take more courses that require prior knowledge of Continuous Time Signals and Systems course content?

1.3 Overview of Study Methodology and Methods

As mentioned before, this study of problematic reasonings used by undergraduate electrical engineering students while engaging with Continuous Time Signals and Systems course content is set in a constructivist framework (Creswell, 2007). A structured protocol was designed to explore problematic reasonings (section 3.3.1) and one-on-one think-aloud interviews were conducted with undergraduate electrical engineering students using the designed protocol (section 3.3.3.4). This study acknowledges the fact that interpersonal communications plays a role in the data collected about the conceptual understanding of any topic using clinical interviews (Montfort, 2011; Säljö, 1999), and so focuses more on the identification of the revealed knowledge (i.e., problematic reasonings) rather than claiming to understand what students know (i.e. misconceptions). The topics for the questions for the protocol were identified through existing literature (Chapter 2) and discussion with Continuous Time Signals and Systems experts (section 3.10). The quality of the protocol was validated by

a) discussion with a total of ten experts in Continuous Time Signals and Systems course content and qualitative research methods (Table 3.5) and, b) the researcher piloting the protocol three times (section 3.3.1). In addition, to validate the protocol, the pilot studies were also used to hone the researcher's skills as an interviewer, and making choices for the most appropriate interview settings (section 3.3.3.4).

Nineteen undergraduate electrical engineering students from Iris University (pseudonym for study site described in detail in section 3.4) voluntarily participated in this study (section 3.6.3). Of the nineteen students, eight had taken only one Continuous Time Signals and Systems course and no subsequent courses, and eleven had taken one or more (up to four) such courses (section 3.6.4). Each student was interviewed individually. The interviews were audio-recorded (section 3.3.3.4). Participants were provided with a calculator, a Fourier transform table, and a related formula sheet during the interviews, in case they needed it (section 3.3.1). The transcribed audio data collected during interviews was analyzed using thematic analysis (section 3.7).

1.4 Rationale

The purpose of this study is to explore the problematic reasonings used by undergraduate electrical engineering students when they engage with Continuous Time Signals and Systems course content. There are very few exploratory studies on engineering students' understanding of concepts similar in nature to the concepts learned in Continuous Time Signals and Systems courses. Therefore, this study has both theoretical and practical significance. Theoretically, this study will add to the literature on conceptual learning in engineering and will help the engineering educators to develop an in-depth understanding of the difficulties faced by students while attempting to access similar content. Furthermore, this study will provide an understanding of the differences of the use of problematic reasonings between students who have taken no additional courses that require prior knowledge of the content of this course and students who have taken the subsequent courses. This understanding will be useful for i) instructors of engineering courses like Continuous Time Signals and Systems to develop successful educational strategies, and ii) engineering curriculum developers to modify overall electrical engineering curriculum to efficiently help students to develop a robust conceptual understanding of Continuous Time Signals and Systems course content. Moreover, the design of this study can be used as an example for future work in understanding students' reasonings within engineering courses.

Although the focus of this study is to understand electrical engineering students' problematic reasonings of the course content, this course is taught in various other engineering disciplines like aeronautics, and astronautics, bio-medical, and mechanical. Therefore, the results of this study can have broader impact beyond electrical and computer engineering education.

1.5 Limitations

There are several limitations of this study.

1. The findings collected from a small sample of volunteer participants from a small private teaching-intensive university may not generalize to all undergraduate electrical engineering students. Additionally, the scope of this study is bounded within the learning experiences of students in one country only.

2. Verbal protocols only reveal knowledge that participants articulate in response to a specific question or a task, and do not reveal all the knowledge possessed by the participants (Säljö, 1999). Therefore, the findings of this study are more representative of what students can verbalize as compared to what they actually know.
3. Although the choices made by the researcher for this research (protocol development, conducting the interviews, data analysis and interpretation, etc.) are validated at all stages with experts in the field of conceptual understanding as well as in the field of signal analysis, the results of this study are not completely free of the researcher's biases.

1.6 Definitions of Fundamental Terms

Following are the definitions of the fundamental terms (arranged alphabetically) used in this study.

Concept - A concept is defined as a piece or cluster of knowledge (Streveler, Brown, Herman, & Montfort, 2014).

Conceptual change - Conceptual change is a process of altering a person's conceptual understanding (Streveler, Brown, Herman, & Montfort, 2014).

Conceptual understanding - Conceptual understanding of a particular topic is defined as beliefs and framework used to acquire new knowledge or perform new applications of old knowledge in that topic (Montfort, Brown, & Pollock, 2009).

Conceptual Knowledge - Conceptual knowledge is understanding of principles governing a domain and the interrelations between units of knowledge in a domain (Rittle-Johnson, 2006, p. 2)

Misconceptions - Misconceptions are "any aspect of an individual's conceptual understanding that resists conceptual change and contributes to an incorrect, naive, or unproductive conceptual understanding" (Streveler, Brown, Herman, & Montfort, 2014, p.1).

Mistakes - Participants' incorrect responses during the interviews for which there is not enough evidence for a problematic reasoning behind them are called mistakes (Definition specifically created for this study and explained more in section 2.5).

Problematic Reasoning - Problematic reasoning is a reasoning that has the potential to hinder conceptual understanding and cultivate misconceptions (Definition specifically created for this study).

Reasoning - Reasoning is a purposeful effort to generate justifiable conclusions and make sense of the problem (Definition specifically created for this study).

CHAPTER 2 - LITERATURE REVIEW

Signals and systems is a core course in the undergraduate electrical engineering curriculum in which students learn about the fundamental concepts of signals and systems and their analyses using various generalized mathematical tools and transforms. Although the concepts discussed in this course are applicable to any signals or systems in the world, signals in this course mainly characterize analog or digital signals representing some analog physical quantities (like audio signals, radio signals, etc.), and systems in this course mainly represent an electrical or electronic systems (like filters, communication systems, or control systems, etc.). This chapter discusses literature on learning Signals and Systems in particular and relevant literature on learning science and mathematics in general.

In the first section of this chapter, I will discuss the course content of Signals and Systems and the difficult concepts that arise out of this subject matter. I will start the section with a discussion of the course content. I will then present the literature on what makes this a difficult course in general followed by specific misconceptions and conceptual difficulties identified in the course. The comparison of course content and literature on difficult concepts presented in this section highlights the gaps in research in conceptually understanding concepts in Signals and Systems, which supports the need for this study discussed in last section of this chapter.

In the second section of this chapter, I will present various pedagogical strategies proposed by engineering educators to improve students' learning in this course. In addition, I will explain the development of the Signals and Systems concept inventory as a tool to evaluate pedagogical techniques and curricular reforms in Signals and Systems. The literature presented in this section acknowledges the numerous attempts of engineering educators to improve students' conceptual understanding of Signals and Systems. The literature in this section will highlight the lack of qualitative evidence to support and validate the success of these pedagogical strategies in achieving students' conceptual understanding. The identification of a lack of evidence to support the current efforts in improving pedagogy of this course will highlight the need for this study that is discussed in the last section of this chapter.

In the third section of this chapter, I will present a few conceptual change and learning theories in science and mathematics. Although there is a pool of information ranging over decades on conceptual change and learning, I will specifically focus on the theories that i) other researchers have used to describe learning hurdles in Signals and Systems (p-prims), and/or ii) have the potential to suggest an explanation for the learning difficulties in this course.

In the fourth section, I will present the findings of the three studies that I conducted at different times on difficulties for students taking Signals and Systems courses. I will first present the findings of my master's thesis about identification of learning hurdles for students taking Signals and Systems. Then I will discuss the potential reasons for these learning hurdles in the light of the conceptual change theories presented in the third section of this chapter. This discussion will help to illustrate the reasons for the choice of theoretical framework for this study. Next, I will present the difficult concepts identified in a quantitative analysis of continuous time SSCI post-test scores of 958 students over a period of ten years at Iris University. In the end of this section, I will discuss the findings from a small study from my *Qualitative Research Methods* class on identification of problems in learning Signals and Systems course content across borders. The literature presented in this section will establish the researcher's understanding of possible difficulties associated with conceptually understanding concepts in Signals and Systems courses.

In the fifth section of this chapter, I will discuss the gaps in the literature on learning Signals and Systems course content as evidenced by the information presented in the previous sections. This will establish the standing of this study in the realm of the research conducted on learning Signals and Systems course content to date.

2.1 Signals and Systems - Course Content and Difficult Concepts

Signals and systems is a core course in electrical, computer, and aerospace engineering curricula, and typically taught in sophomore or junior year (Wage & Buck, 2001). Standard textbooks for this course are *Signals and Systems* (Oppenheim, Willsky, & Nawab, 1997) and *Linear Systems and Signals* (Lathi, 1998), (Wage, Buck, Welch, & Wright, 2002a, 2002b), however, engineering educators have been using various other books as well (Kanmani, 2011). Course content mainly focuses on continuous time signals and continuous time systems (Wage & Buck, 2001). A thorough understanding of this course is important in the field of electrical engineering because concepts learned in this course are pre-requisite concepts for many core courses in the undergraduate electrical engineering curriculum like circuit analysis, communications, and control systems as well as many specialized courses like digital signal processing.

2.1.1 Course Content

As mentioned in section 2.1, Signals and Systems is a core course in a typical undergraduate electrical engineering curriculum and is a pre-requisite for many core courses in the undergraduate electrical engineering curriculum like circuit analysis, communications, control systems and specialized courses like digital signal processing, etc. (Munson & Jones, 1999). The three major content areas in typical Continuous Time Signals and Systems courses are i) Signal representation and operation, ii) Frequency analysis, and iii) System Analysis. Signal representation and operation content area comprises topics like representation of signals using mathematical equations and graphs, components of signal (even, odd, etc.), types of signals, various operations on signals like time shifting, time scaling, etc, complex signals like Dirac delta, sinc, unit step function, etc. Frequency Analysis content area comprises of analysis of signals through Fourier series and transform. System Analysis content area covers topics such as different types of systems with emphasis on linear time-invariant systems, impulse response, and LTI system analysis through convolution and Laplace transform (Evans, Karam, West, & McClellan, 1993; Munson & Jones, 1999; Wage, Buck, Welch, & Wright, 2002).

2.1.2 Problems in Learning

The three major content areas in the *Signals and Systems* courses are Signal properties, Fourier analysis, and system analysis (Laplace transform, and convolution) (Evans, Karam, West, & McClellan, 1993; Munson & Jones, 1999; Wage, Buck, Welch, & Wright, 2002). Some studies have suggested that the abstract nature and disconnection of these concepts from daily life could make them difficult to understand. Additionally, these concepts and their applications in the physical world are described through mathematics, which requires students to combine advanced mathematical concepts with their perception of physical systems (Nasr, Hall, & Garik, 2005). Consequently, a large part of this course deals with abstract mathematical constructs. A few studies have contended that these abstract mathematical constructs are difficult to visualize and comprehend (Shaffer, Hamaker, & Picone, 1998; Nasr, Hall, & Garik, 2005, 2007; Tsakalis et al., 2011). For conceptual understanding of any subject matter, students often need to know the usefulness of what they learn and want to be sure that the information they acquire is useful in daily life (Çetin, 2004). Nasr, Hall, and Garik (2005) have argued that the disjointed-from-everyday-life nature of concepts in Signals and Systems course content makes this course different from other courses in engineering, like Electronics and Circuit Analysis.

In addition, for better understanding of this course, sophisticated mathematical skills rather than just knowing formulas and carrying out fixed procedures to solve the problems are deemed necessary in most of the studies. There is sufficient anecdotal support that, these days, engineering students either lack the mathematical proficiency required to solve any problem in physics or engineering, or fail to apply their knowledge of the mathematics to any physics or engineering context (Nasr, Hall, & Garik, 2005). Bruner (1962) argues that students can find difficulty in understanding mathematical concepts if they cannot understand them intuitively or be able to translate intuitive ideas into mathematics. Betz (1978) suggests that math anxiety in engineering students can influence their understanding of math-influenced engineering concepts. Moreover, in the university level engineering education, the gap between application-oriented expectations of students and theory-focused lectures is claimed to have a considerable effect on the motivation of students (Munz, Schumm, Wiesebrock, & Allgower, 2007).

There is a limited amount of work done in conceptual understanding of topics taught in Signals and Systems courses. I have divided this literature into three categories based on the particular course content investigated in each study. The categories are: i) Linear-time-invariant system analysis and convolution and, ii) mathematical concepts and thinking. The difficult concepts discussed in one category may not be uniquely attributed to the problems in learning that particular concept, but rather a combination of more than one category of difficult concepts.

2.1.2.1 Linear-Time-Invariant System Analysis and Convolution

The difficulties in learning system analysis and convolution identified in the previous literature are as follows:

i. Nasr, Hall, and Garik (2007) used the concept of DiSessa's p-prims (fundamental knowledge structure) and coordination classes (large and complex knowledge structures composed of combination of p-prims) to explain the faulty cognitive resources underlying the mathematical reasonings of students attempting to learn continuous time (CT) linear, time-invariant (LTI) electric circuits. They were specifically interested in finding reasons for conceptual problems in the context of LTI circuits, as the students in aerospace engineering program at MIT are taught Signals and Systems courses in this context. For this purpose, they interviewed 51 students enrolled in Signals and Systems course in the Department of Aeronautics at Massachusetts Institute of Technology, in 2002-2003. Their results suggested that the faulty reasonings of the students when engaging with topics related to superposition, convolution, and Laplace transform are mostly because of the inappropriate invocation of the interval matching readout strategy. In other words, they argued that the students employ the interval matching strategy in problems where its use is inappropriate. Readout strategies, as presented by DiSessa (1983, 2002) are part of a large complex knowledge system called coordination class, which is an integrated model of numerous smaller knowledge structures that result in an expert-like understanding of a certain scientific concept. Readout strategies constitute the ways in which a particular concept or a situation is observed or understood (DiSessa, 1983, 2002).

ii. In a follow-up study, Nasr, Hall, and Garik (2009) investigated naive reasoning of aeronautical engineering students related to the concepts of linearity, time-invariance, and convolution, to provide a foundation for designing effective instructional materials for Signals and Systems courses. They suggested that their findings would help in designing a better pedagogy for this course as the knowledge of students' skills and pre-conceptions is necessary for effective pedagogical design (NBPTS, 2005). In addition to interval matching, symmetry invocation was also claimed to be a commonly employed faulty naive reasoning. Symmetry invocation as defined by Nasr, Hall, and Garik (2009) is students' undue bias to apply symmetry properties to analyze all the systems including non-symmetric systems.

iii. Wage, Buck, and Hjalmarson (2006a) conducted semi-structured interviews with nine students and they argued that in some instances the "connotations" of the daily use of the term "filter" limits the student's understanding of the concept of scaling factor in the concept of "filters as systems" taught in this course. According to them, students face difficulties in connecting the concept of a scaling factor to a filter as their perception of filters adheres to the everyday use of filter such as air filter, coffee filter, or spam filter. The difficulty in learning a new concept or term about which the students have prior familiarity in a different meaning is suggested in other studies and contexts in science education as well. These include Herman, Kaczmarczyk, Loui, and Zilles's (2008) study on computer science and computer engineering students' misconceptions in logic design concepts, and DiSessa, Gillespie, and Esterly's (2004) study on the K-12 students' concepts of force.

iv. Wage, Buck, and Wright (2004) have argued that students face difficulty in relating the concepts of impulse response and complex frequency analysis (Laplace transform) to analyze a real system (Wage, Buck, & Wright, 2004).

2.1.2.2 Mathematical Concepts and Thinking

1. Wage, Buck, and Wright (2004) have used the Signals and Systems concept inventory (SSCI) (to be discussed in detail in section 2.2.1) to illustrate that a sound mathematical knowledge is helpful in understanding the concepts in Signals and

Systems course content. Although the reliability of an SSCI as an assessment instrument is not yet established, ever since its initial design in 2001 it has been widely used in over twelve schools for research in understanding problems encountered by students taking Signals and Systems courses (Wage, Buck, & Wright, 2004). Without the information about the reliability of an instrument, the consistency of the results of any research using that particular instrument remains questionable (Gilbert, 1989). However, in an interest to include all the discussion in the literature about possible obstacles in conceptual understanding of Signals and Systems course content, I am presenting the results of a study conducted using an SSCI (Wage, Buck, & Wright, 2004) that claims that the mathematical understanding of students contribute towards conceptual learning of concepts covered in Signals and Systems courses.

a) The study claimed a positive correlation of the gain in SSCI scores of the students with their grades in some prerequisite courses (calculus, differential equations, and circuits) within the curriculum of electrical and computer engineering.

b) Wage, Buck, and Wright (2004) argued, based on students' responses in SSCI pretests and posttests, about the presence of three persistent misconceptions in students. Firstly, they suggested that students incorrectly believed that the real impulse response corresponds only to systems with real poles and zeros. Secondly, they claimed that the students incorrectly thought that the multiplication in the time domain corresponds to multiplication in the frequency domain as well. Thirdly, they asserted that the students falsely believed that a frequency response with two resonant peaks have one pole in the left-half plane and one in the right-half plane, that is, they mistakenly reverse the roles of the real and imaginary axes of the pole-zero plot.

c) A few studies have suggested that the students face difficulties in understanding the need and importance of transforms, which further confuse them to connect alternate shapes of the same signal in different domains (Wage, Buck, & Wright, 2004; Buck & Wage, 2005; Wage, Buck, & Hjalmarson, 2006a).

2. Nasr, Hall, and Garik (2009) have argued that the students find difficulties in doing convolution by graphical method. They suggest that while performing convolution by graphical method, students demonstrate problems in solving long integrals, multiplying two signals, putting appropriate limits, defining signals piece-wise, and flipping and shifting the signal. They further claimed that the difficulty in doing convolution was more significant when one of the two functions being convolved had any of these characteristics: (i) did not begin at t=0, (ii) was piece-wise, (iii) was non-causal, and (iv) had negative values over a certain interval of time (Nasr, Hall, & Garik, 2009).

3. Nelson, Hjalmarson, and Wage (2011) used two types of in-class assessments: group exercises and individual exams to observe students' understandings of Signals and Systems course content. They claimed that the mathematical areas where students exhibited significant gaps in their knowledge were i) definitions and/or evaluation of the conditions of causality and stability of a system, ii) mathematical representation of signals and systems as either a function or a graph, iii) different types of independent and dependent variables together in a function, and v) impulse response.

2.2 Pedagogical Strategies for Signals and Systems Courses

Engineering educators have spent a great deal of effort on developing effective ways to teach Signals and Systems courses. So (2012) collected feedback from students; gathered quantitative data from student evaluations and grade distributions; and concluded that "chalk-and-talk" lecturing style is a preferred way to teach this course instead of using PowerPoint slides. Hanselman (1992) based his research on learning styles (Felder & Silverman, 1988) of an engineering student and proposed to teach continuous-time concepts before discrete-time concepts in this course. According to him, teaching continuous-time concepts before discrete-time concepts will support inductive progression of the course content, attend to the need of the students who learn through sensing, and help both global and sequential learners.

Many engineering educators have proposed to teach this course using computer tools like excel (Stanton, Drozdowski, & Duncan, 1993) and MATLAB (Cavicchi, 2005; Guan, Zhang, & Zheng, 2009; Han, Zhang, & Qin, 2011) to help students bridge the gap between the abstract nature of the concepts in this course and their real life applications. Stanton, Drozdowski, and Duncan (1993) proposed that computer exercises using spreadsheets are more effective than structured languages in reinforcing students' fundamental concepts taught in signals and systems courses, specifically Fourier series analysis, convolution of finite duration signals, and state-space solutions to linear circuits. Cavicchi (2005) presented a set of experiments integrating concepts like sampling, aliasing, system modeling, frequency response, discrete Fourier transform, power spectrum, correlation, and auto-correlation in MATLAB that explain, predict, and evaluate various measurements. He surveyed all eight students who worked on the suggested lab sequence over the two years and the data from students suggested that these labs helped all the students but high-achieving students got more out of it.

Moreover, some educators have recommended hands-on techniques to improve students' understanding of this course. These include hardware-based signal processing laboratory exercises to enhance students' understanding of signal processing concepts (Huettel, 2006), and the use of inexpensive and portable LEGO MINDSTORMS NXT platforms for signal processing experiments (Ferri et al., 2009). Huettel (2006) recommended four hardware-based signal processing laboratory exercises to enhance students' understanding of signal processing concepts. These four exercises covered real-time audio effects, dual-tone multi-frequency, sampling and aliasing, and voice-scrambler-descrambler. He piloted the lab on his students and administered an anonymous survey at the end of semester about students' experiences in the lab. The results of the survey illustrated a clear understanding among students about sampling and aliasing and about real-world applications of concepts covered in this course. In addition, the survey results showed an increase in students' level of interest in the field of signal processing. To help students get a practical experience of the abstract and mathematical concepts in this course, Ferri et al. (2009) proposed a set of inexpensive signal processing experiments for undergraduate students in electrical and mechanical engineering based on the LEGO MINDSTORMS NXT platform. The set-up of the experiments was portable, relatively inexpensive, and rugged enough that students could perform them at home, as well as in the classroom. These experiments highlighted basic concepts in Signals and Systems course content like sampling, aliasing, digital filtering, frequency analysis, system identification, and control design.

Additionally, Simoni (2011) has developed a hardware platform that provides hands-on experiences to undergraduate electrical engineering students in learning continuous-time signals and system course content. The hands-on experiences are expected to improve students' understanding and interest in frequency domain concepts. The hardware platform facilitates students to work with a wide variety of realistic and personalized signals including an audio signal (through a microphone), a voltage signal, and an ECG signal (through an instrumentation amplifier). The hardware platform can perform various operations including multiplication, addition, filtering, and sampling on the different input signals. The platform allows students to manipulate realistic continuous-time systems and observe corresponding input and output signals simultaneously in the time and frequency domains.

2.2.1 Signals and Systems Concept Inventory

A concept inventory (CI) is an assessment tool that may be administered as pretest and posttest in a course and is often used to measure gain in conceptual understanding of a learner (Wage & Hjalmarson, 2006b; Buck, Wage, Hjalmarson, & Nelson, 2007). The design of any CI is based on the knowledge collected by the developer(s) about commonly held misconceptions of students in a particular discipline (Evans et al., 2002). The CIs use misconceptions as distractors to identify if a student is able to recognize the correct answer out of the common misconceptions (Evans et al., 2002). Streveler et al. (2011) have presented an efficient methodology for creating valid and reliable concept inventories to measure students' misconceptions in engineering and science domains. They suggest that successful concept inventory design involves aligning the three corners of the assessment triangle, i.e., cognition, observation, and interpretation. The cognition corner corresponds to the identification and validation of important concepts (can be done through Delphi studies), the observation corner corresponds to the development and pilot of the inventory and the interpretation corner includes establishing the instrument reliability.

The Signals and systems concept inventory (SSCI) was initially developed in 2000 for the curriculum of this course within electrical and computer engineering (Wage & Buck, 2001). SSCI is a 25 question multiple-choice exam devised to measure students' understanding of basic concepts in an undergraduate Signals and Systems course (Wage & Buck, 2001). In just a few years, SSCI was already used on over 1000 students in 12 schools (Wage, Buck, & Wright, 2004). Since the development of SSCI, investigations have continued to analyze the results of SSCI in undergraduate courses to assess students' performance in Signals and Systems courses from year to year, and to identify concepts that are difficult for most of the students so that the future offerings of this course can be improved (Ogunfunmi, 2011). There is no evidence in previous studies about the development of the cognition or interpretation corners of the SSCIs developed so far, which, I argue, presents a doubt in the use of SSCIs and interpretation of students' misconceptions based on the SSCI scores. I contend that the important concepts that need to be covered in any CI cannot be determined without the development of the cognition corner and the results of any CI test would be doubtful if the interpretation corner is not established (Streveler et al., 2011).

Wage and Buck (2001) initially designed the questions in the SSCI to focus on core concepts of this course including linearity, time-invariance, impulse response, convolution, Fourier analysis, Laplace transform, representations of systems with linear differential equations, pole-zero diagrams and their relationship with impulse and frequency responses of systems, filtering, and stability. All the questions were designed in a way that they need minimal or no mathematical computations (Wage & Buck, 2001). These core concepts were further divided into six categories: pre-requisite mathematical concepts, linearity and time-invariance, convolution, transforms, filtering, and sampling (Evans et al., 2003). Each question in SSCI had four options and the three incorrect choices, also called distractors, were claimed to be designed to capture students' common misconceptions in Signals and Systems course content (Wage, Buck, & Hjalmarson, 2006a). The processes used in the development of SSCI to i) design the distractors to capture students' misconceptions, ii) gain knowledge of students' misconceptions to design the distractors, and iii) design the questions so that no mathematical computations are necessary are not known. There are distinct versions of the SSCI tests for continuous time (CT) and discrete time (DT) concepts. The SSCI website (http://signals-and-systems.org) is maintained to provide information about its ongoing study.

Validity of any instrument is an important criterion to determine the worth of the results obtained by using that particular instrument. Validity of an instrument is usually established by correlating the scores obtained by the instrument with some similar scores. The validation of SSCI was initially done in 2002 (Wage, Buck, Welch, & Wright, 2002). The cumulative GPA, Signals and Systems course grade, and other prerequisite courses of 174 students were correlated with their SSCI scores. These students were from four different schools, George Mason University, US Air Force Academy, the US Naval Academy, and University of Massachusetts Dartmouth. The correlation results of SSCI posttest scores and grades were found consistently significant in students across all four campuses. In addition, analysis of variance (ANOVA) test was performed on the SSCI scores to check for gender and racial bias. The results showed no statistically significant correlations between males and females or between whites and under-represented minorities, hence supporting the validity of SSCI. Buck, Wage, Hjalmarson, and Nelson (2007) validated SSCI by the results of two different analyses. Firstly, they claimed a statistically significant correlation between SSCI scores and the final exam scores for questions on convolution and Fourier transform properties. Secondly, they interviewed 18 students about questions on frequency-selective filtering and convolution, and correlated their interview responses with their SSCI scores in questions related to these concepts. Their results suggested that students' understanding of both time-frequency relations and convolution was well-connected to their performance in related questions in SSCI (Buck, Wage, Hjalmarson, & Nelson, 2007).

Wage, Buck, and Hjalmarson (2006a) used SSCI to probe student's understanding of frequency selective filtering in Signals and Systems courses. They interviewed students and used their responses to provide insight into the conceptual models that students employ to reason about frequency and filtering. Their results claimed that students' major misconceptions revolved around magnitude and phase of the frequency response and the relationships between the magnitudes of the time domain signals and their spectra. Based on the findings of this study, Wage, Buck, and Hjalmarson (2006a) proposed to include questions in SSCI that would probe students' understandings of filtering and comparison of magnitudes of a signal in both the time and frequency domains. Additionally, in the same study, Wage, Buck, and Hjalmarson (2006b) provided a summary of students' SSCI pretest and posttest scores to claim gains in students' conceptual understanding of Signals and Systems course content. Furthermore, students' scores in SSCI pretest and posttest were used to compare interactive-engagement and traditional lecture based approach to teach this course. It was contended that at the end of the semester, students' gain in knowledge was 39% more than their knowledge at the start of the semester, when taught through interactive classes, whereas, the gain in students' knowledge through traditional lecture-based approach was only about 22% (Wage, Buck, & Hjalmarson, 2006b).

Additionally, since its development, SSCI has been used as a tool to evaluate Signals and Systems curriculum and pedagogical techniques. In one study, both SSCI pre-test scores and SSCI gains were correlated with grades in several prerequisite courses (calculus, differential equations, and circuits) within the curriculum of electrical and computer engineering. The results demonstrated that students learned this course better when they had adequate background and expertise in mathematics (Wage, Buck, & Wright, 2004). SSCI was also used to show that active and cooperative learning (ACL) instructional format facilitates students' learning of signals and systems course content (Buck & Wage, 2005).

Padgett, Yoder, and Forbes (2011) proposed six extended applications of SSCI that were not in the scope of the original design of SSCI. These were (i) comparison of how differences in international educational style impact conceptual learning, (ii) introduction of conceptual/graphical testing to international instructors, (iii) demonstration of student outcomes for ABET assessment, (iv) comparisons of reinforcement of concepts in follow-on electives, (v) longitudinal studies, and (vi) comparison of variation of student performance with student learning style. Students' SSCI scores were claimed to be useful for the identification of the differences in international curricula, faculty mindset, and curriculum assessment. Additionally, SSCI scores were argued to be useful for determining the extent to which the subject matter of Signals and Systems was reinforced in subsequent elective courses and the improvement in students' understanding of the fundamental concepts taught in Signals and Systems courses over time (Padgett, Yoder, & Forbes, 2011).

SSCI has been used in many universities and some research has been done in the past on the validation of the SSCI (discussed in the beginning of this section), not much research is conducted in determining the reliability statistics of SSCI. The lack of this important information introduces ambiguity in the results of any research in assessing students' misconceptions in Signals and Systems courses using SSCI. This is a limitation of using an SSCI as a quantitative assessment tool.

2.3 Conceptual Change and Learning Theories

Much research has been done in conceptual learning in science and mathematics since early 1970s (Smith, DiSessa, & Roschelle, 1994). Piaget (1971), one of the early cognitive development theorists, believed that cognitive development is an active construction process in which children increasingly build their own knowledge according to their biological tendencies. Vygotsky (1962) on the other hand highlighted the importance of the role of a teacher in child's learning and suggested that learning actually happens in the zone of proximal development. The term conceptual change was first introduced by Thomas Kuhn (1962) to explain that the concepts embedded in a theory change their meaning when theory changes. According to him, the conceptual systems of a learner are well integrated and oppose bit-by-bit change. Therefore, science changes in a revolutionary manner.

Learning theories can be domain general or domain specific. Domain general theories focus on principles and mechanisms that can describe all aspects of learning, for example, Piagetian and Vygotskian learning theories. Domain specific theories focus on the description and explanation of conceptual changes within specific content of knowledge. Domain specific theories compliment domain general theories and yet allow researchers to flexibly make hypothesis about the way a specific content is structured (and re-structured), without necessarily committing to general constraints or modules (Vosniadou, Vamvakoussi, & Skopeliti, 2008).

In this section, I will describe three learning theories that were helpful in the past to explain numerous misconceptions and difficulties in learning science related concepts. I will use these theories later in this chapter to suggest some explanation for difficulties in learning 'Signals and Systems' course content. These are i) Ontological categorization, ii) Framework theory, iii) P-prims. In the end of this section, I will discuss conceptual change theories in mathematics in general, and 'advanced mathematical thinking' in particular.

2.3.1 Ontological Categorization

While Kuhn (1962) and many other philosophers of science, like Lakatos (1970) and Laudan (1978) discussed theory of conceptual change in terms of a paradigm shift, Chi and colleagues (1992, 1993, 1994, 1997, 2002, 2008, 2012) presented the idea of conceptual change as a change in the categorical status of a concept. This theory emphasized that assigning correct categories for novel concepts was important for conceptual understanding. This was based on the idea that the novel concept would automatically inherit features and attributes from its category membership and the learner could use knowledge of the category to make many inferences about the newly learned concept. Chi and colleagues argued that the misunderstandings would happen when the learner would place a new concept in an incorrect category. Moreover, robust misunderstandings would occur if the new concept is placed in an incorrect lateral or ontological category because laterally or ontologically different categories had distinct and mutually exclusive properties. Chi (1992, 2008) defined ontological categories as lateral categories between different conceptual trees that do not even share any common super-ordinate level categories.

With reference to the idea that categorization allows new concepts to inherit categorical properties, Chi (2008) has suggested two possible mistakes within categorization and learning: hierarchical and categorical. A hierarchical mistake claims that a learner fails to recognize an obvious basic category of a new concept or phenomenon and assigns it to a more general hierarchical category. A categorical mistake claims that a learner fails to recognize the category of a new concept or phenomenon and assigns it to an incorrect lateral or ontological category. Chi (2008) argued that the category mistakes, unlike hierarchical mistakes, are more damaging to conceptual understanding of new concepts because the new concept inherits all the attributes of the erroneously assigned lateral category.

Category mistakes are not only claimed to be in conflict with the correct scientific knowledge but they are also claimed to be robust, i.e. they are suggested to be persistent and resistant to conceptual change (Chi, 1993, 2005, 2008; Chi, Slotta, & de Leeuw, 1994). Chi argues that to correct a robust misconception and overcome the barrier of conceptual change across lateral and ontological categories, a learner must have the knowledge about the possible categories to which a concept can belong, and be able to confront the knowledge at the categorical level to identify if there is a category mistake. A few category mistakes identified in the science education literature so far are force, temperature, heat, light, electric current, and diffusion (Chi, 2008; Chi, Roscoe, Slotta, Roy, & Chase, 2012).

2.3.2 Framework Theory

Vosniadou and Vamvakoussi (2006) presented the theory that children start to form their knowledge frameworks from birth and keep enriching the initially formed frameworks whenever they encounter any new knowledge. They argued that this makes a human's knowledge system a complex integration of a person's numerous beliefs developed by interaction with physical, social, and cultural worlds around him/her. They argued that the assimilation of a novel knowledge into an existing knowledge system becomes a problem when the person does not have a compatible initial framework to add this new knowledge and is not even aware of the missing appropriate framework.

The framework theory approach suggested that when a learner encounters some new information and has no compatible initial framework to assimilate it, misconceptions occur. This is why, according to this theory, mathematical concepts that are contradictory to the learner's prior everyday knowledge are harder to learn. Vosniadou and Vamvakoussi (2006) argued that to learn concepts that conflict with well-acquired everyday knowledge, the learners are needed to be actively aware of their existing knowledge structure, and to be able to reassess their existing knowledge structures whenever required. This ability to be actively aware of one’s knowledge corresponds with the metacognition model of cognitive learning theory (Svinicki, 1999). Additionally, the ability to reassess the knowledge schema corresponds to structuring and restructuring of memory discussed in the early cognitive model of learning (Svinicki, 1999).

Vosniadou and Vamvakoussi (2006) based their framework theory on empirical studies conducted on sixteen ninth-graders in a middle class school in the Athens area. Students participated voluntarily in this study and they all had varying mathematics grades in class as reported by their teacher. Each student was interviewed individually for about an hour for this study. The questionnaire used for the interviews was developed after conducting an experimental pilot study on a larger pool of questions related to the concepts of rational numbers and fractions. The objective of this study was to find out if students hold on to their prior beliefs in learning mathematics, just as they do in learning science concepts (Vamvakoussi & Vosniadou, 2004).

The results of this study showed that the students exhibited a lack of understanding of rational numbers. The study also claimed that students demonstrated confusion in understanding how many numbers, either finite or infinite, fall between two rational numbers. In addition, the study argued that the students showed weak understanding of similarity between a fraction and a decimal number (Vosniadou & Vamvakoussi, 2006). Vosniadou and Vamvakoussi (2006) discussed that the misconceptions identified in their study could be explained based on framework theory and claimed that the radical re-structuring of the initial belief from everyday life about numbers is required to properly understand the concept of a rational number.

One drawback of Vosniadou and Vamvakoussi's (2006) framework theory is that this theory implies that the initial frameworks in the mind of a person are coherent. This can be true in some cases, but not always. A counter example is presented by DiSessa and colleagues (DiSessa, Gillespie, & Esterly, 2004; DiSessa, 2008).

2.3.3 P-prims

While Chi talks about the category mistakes, others explore the role of reasoning resources in student's learning processes. Reasoning resources are described as the cognitive raw materials that constitute the knowledge used to describe and explain thinking and reasoning. They included basic knowledge elements (Redish, 2004), mathematical knowledge structures (Sherin, 2001a, 2001b), interpretive strategies, and cognitive nets of smaller reasoning resources called p-prims (DiSessa & Sherin, 1998; DiSessa, 2002).

DiSessa et al. (1982, 1983, 1988, 1993, 1994, 2008) presented a constructivist approach to conceptual learning that was based on students' intuitive knowledge. They dismissed the ideas that i) the students' prior knowledge conflicting with the expert's knowledge was a misconception, and ii) conceptual change referred to correcting the misconception. Instead, they proposed that the student's prior knowledge contradicting with expert's knowledge was not a misconception but a more fundamental and abstract cognitive structure called phenomenological primitive or p-prim.

DiSessa (1982) defined p-prims as phenomenological as they originated automatically from interpretations of some experienced reality and primitive because they were assumed to be self-evident and not be further reduced to smaller cognitive structures. Any situation within their span of applicability would activate them. DiSessa (1982) claimed that one might liken p-prims to Piagetian concrete operational understanding. P-prims are claimed to facilitate learning if they are activated in an appropriate situation, and improper activation of p-prims in any situation is argued to lead to misconceptions (DiSessa, 1993, 2002; DiSessa & Sherin, 1998).

For example, in a study, students were asked to explain why weather is hot in summer, and most of them replied that it is because the earth is closer to the sun in summer. It was suggested that the participants invoked the p-prim connecting proximity and intensity (closer means stronger) that is correct information but was activated incorrectly in interpreting the hot weather in summer. Hammer (2000) argued that this p-prim closer means stronger was not a misconception and would have helped in knowledge empowerment if activated appropriately in a situation (Hammer, 2000).

DiSessa (2008) emphasizes that p-prims are important in everyday learning, and are often if not always productive in everyday thinking. He claims that the inappropriate contextual use of otherwise correct p-prims explains the robustness of the difficulty in conceptual change (DiSessa, 2008). In contrast to the general argument that conceptual change requires erasing the previous concept (Kuhn, 1962), DiSessa (2008) contends that conceptual change requires corresponding p-prims to be re-contextualized and not erased. Many such coordinated contextual changes of p-prims create normative scientific concept (DiSessa, 1982, 2008).

Based on the idea that coordinated changes in the contextuality of many corresponding p-prims create conceptual understandings, DiSessa (1983, 1988, 2008) also argued that the knowledge structure of a learner is fragmented in contrast to the idea of coherent naive knowledge of a learner (Kuhn, 1962; McCloskey, 1983; Carey, 1999; Vosniadou, 2002). He called it "knowledge in pieces." According to him, the real issue in conceptual learning is the grain size at which we describe conceptual structure instead of coherence or "knowledge in pieces."

2.3.4 Learning and Conceptual Change Theories in Mathematics

The research in conceptual change approach in mathematics learning and teaching started more recently. Mathematics was considered different from physical sciences, and so for a long time, the mathematics education community was hesitant to espouse conceptual change theories developed primarily in the context of physical sciences (Vosniadou, 2008). Kuhn (1962) suggested that since mathematics is based on deductive proofs and not on experiments, it must be separated from the pattern of scientific development and change. Additionally, unlike science, new theories in mathematics usually carried mathematics to a more general level of analysis and enabled a wider perspective, which created possibilities for new solutions (Dauben, 1984; Corry, 1993). However, Vosniadou (2008) contends that from a learning point of view, students' experiences remain the same across mathematics and science. Additionally, students develop naive mathematics from everyday experience just as they develop naive science, which may facilitate or hinder learning. Vosniadou, (2008) uses such similarities to support the argument that the conceptual change theories for science can be successfully applied in the case of mathematics learning.

I will next present the discussion between conceptual vs. procedural knowledge that helps to understand the learning of engineering concepts that involve mathematical modeling and abstractions. Later, I will discuss advanced mathematical thinking skills required for understanding such engineering concepts followed by Lesh's translational model.

2.3.4.1 Conceptual vs. Procedural Knowledge

Numerous theories on learning acknowledge the difference between conceptual and procedural knowledge (Bisanz & LeFevre, 1992; Anderson, 1993). Procedural knowledge corresponds to a person's understanding to execute certain action in a sequence to solve problems. This knowledge is specific to a specific problem and is usually not generalizable. On the other hand, conceptual knowledge corresponds to a person's implicit or explicit understanding of the principles within a domain and of interrelations between units of knowledge in a domain. Conceptual knowledge is flexible, not specific to a specific problem, and generalizable. Most studies on conceptual versus procedural knowledge are focused on deciding which type of knowledge develops first. Conceptual-first theories advocate that conceptual knowledge in a domain is developed first and later used to develop procedural knowledge in that domain (Halford, 1993; Geary, 1994; Gelman & Williams, 1998). On the contrary, procedures-first theories advocate that the learner starts with learning procedures for solving problems in a domain that then helps to create conceptual knowledge within that domain (Fuson, 1988;

Karmiloff-Smith, 1993; Siegler & Stern, 1998). Additionally, there is third model for the development of conceptual and procedural knowledge, an iterative model, which suggests that both types of knowledge develop iteratively, gain in one type of knowledge furthers gain in the other type of knowledge and the cycle continues, irrespective of which comes first (Rittle-Johnson, Siegler, & Alibali, 2001).

Vincenti (1990) claimed that one of the key epistemological features that distinguish engineering from science is that engineers work with models and abstractions of concepts. Steif (2004) contends that many concepts within various models and abstractions can only be understood from the procedures of the models, which poses a methodological concern to distinguish increase in procedural knowledge from seemingly increase in conceptual knowledge (Steif, 2004). Additionally, it is argued that students hold an incorrect understanding of the role of mathematics in physics. They can perform mathematical operations correctly in the context of a mathematics problem yet remain unable to perform the same operations in the context of a physics problem (Steinberg, Saul, Wittmann, & Redish, 1996). Streveler, Brown, Herman, and Montfort (2014) contend that distinguishing conceptual knowledge from procedural knowledge is important to suggest improvement in the curriculum by identifying the most important knowledge and ways of thinking.

2.3.4.2 Advanced Mathematical Thinking

A large amount of research has been done to observe how students learn mathematics and many suggestions have been presented to improve learning of mathematical concepts so that students can apply the learned mathematical knowledge constructively in practical applications. The range of this research includes discourse analysis (Jamison, 2000), talk-in-action (Chapin, O'Connor, & Anderson, 2009), virtual reality (Roussou, 2009), inquiry communities (Jaworski, 2004), patterns (Steen, 1988), metacognition (Schoenfeld, 1992), problem-solving (Schoenfeld, 1992), informal learning (Civil, 1990), shift of attention (Mason, 1989), analogical reasoning (English & Sharry, 1996), cognitive apprenticeship (Brown, Collins, & Newman, 1989), and simulations (Snir, Smith, & Grosslight, 1993). All these theories are important and helpful in learning mathematics. Advanced mathematical thinking theory (Dreyfus, 1991) suggests explanations for i) students' difficulties in learning engineering concepts taught through mathematical concepts and models, and ii) how these difficulties can be removed.

According to Dreyfus (1991), solving a simple integral like $\int_{-\infty}^{t} sin\omega t\, u(t)e^{-j\omega t}dt$ involves a large variety of simultaneously interacting mental processes like graphical visualization of each function in the integration, visualization of the area under the graph of the product, translation of the product into a simpler function, performing integrals, checking and generalizing the answer in different representations, etc. He suggested that sometimes even when the students are able to perform standardized procedures using defined formulas, as a computer would do; they lack the skill to use their mathematical knowledge more flexibly in unknown situations. He argued that in such situations even a slight change in the structure of the problem or formula could block students' mental processes (Dreyfus, 1991).

Dreyfus (1991) discusses the importance of psychological processes in mathematical learning and suggests that students learn advanced mathematical concepts through reflection on mathematical activity. According to him, the reason for students' failure to reflect on advanced mathematical concepts is the incomplete presentation of these concepts. He says that mathematics is created from many abstractions and assumptions but teachers and mathematicians present mathematics to students in the refined form. This simplified presentation of otherwise complex and abstract mathematical concepts hinders the creation of mental processes required for advanced mathematical learning. Additionally, although abstract mathematical concepts are powerful in obtaining generalizations in mathematical concepts, they do not have any specified intrinsic properties, and can be understood only in terms of their relationships with other similar or different concepts. This makes them hard to intuit. Therefore, Dreyfus (1991) argues that understanding abstract concepts requires a mental ability to shift attention from the concepts themselves to the structure of their properties and relationships with other concepts. He claims that the knowledge of mathematical concepts without sophisticated mental processing skills can enable students to apply mathematical formulas in well-structured questions, but does not prepare them to reflect on mathematical activities. Dreyfus (1991) emphasizes that students can only learn advanced mathematical concepts through mental processing (like visual imagery and metacognition) of these concepts. Additionally, little knowledge is gained about physics concepts when they are learned through mathematics because the concepts remain very abstract (Bruner, 1962).

Dreyfus (1991) has used a descriptive approach for this study. He has backed up his idea based on the results of the previous empirical studies, which include a study on school children in understanding their calculus knowledge (Selden, Mason, & Selden, 1989), a study on the instructional techniques leading to misconceptions in students (Davis, 1988), and a study on improvement in mathematical knowledge of school children using graphical tools (Ruthven, 1990). He used results from all of these studies to prove his claim that advanced mathematical learning happens through active mental processing and metacognition. According to Dreyfus (1991) students can only learn advanced mathematical concepts through the conscious interaction of a large number of mathematical and psychological processes in their minds. These processes include representation, translation, modeling, generalizing, synthesizing, etc. I will explain some of these processes.

Representations correspond to creating a mental representation or visualization of a mathematical concept, such as, graphs, algebraic formulas, flow diagrams, and tables. Understanding the representation of a concept is necessary for learning a concept, particularly when the same concept has seemingly contradictory multiple representations. The richer the mental representation created by a learner about a particular concept is, the better the concept is learned. The ability to achieve a rich mental representation corresponds to the ability of a learner to link multiple aspects of the same concept in his/her mind, which is only possible when the learner is consciously aware of these representations (Dreyfus, 1991). This looks like the metacognition model of learning (Svinicki, 1999).

In addition to creating a mental representation of a concept, learning an advanced mathematical concept requires an ability to switch mentally from one representation or formula of a concept to its other representation or formula. This is called switching representations or translation. For example, a trigonometric function has multiple representations and properties, like frequency, amplitude, phase, graph, table, points of extremas, and zeros. Students get overwhelmed when they encounter many representations of the same concept and try to stick to only one mental image, which leads to misconceptions (Dreyfus, 1991).

Moreover, learning an advanced mathematical concept requires an ability to mentally translate the mathematical concept in different contexts. This corresponds to the ability of a student to apply a mathematical concept to multiple problem statements. For example, applying a second-order, linear differential equation to an electric circuit problem requires students to translate the same quantities in the formula in the context of the differential equation as well as in the context of the electric circuits (Dreyfus, 1991). Furthermore, modeling refers to a mental mathematical model of a physical system. Generalizing corresponds to the ability of a learner to relate a mathematical concept learned in one situation and apply it to other situations by identifying commonalities between different situations. Synthesizing corresponds to the ability to interrelate many mathematical concepts into one single big picture (Dreyfus, 1991).

Dreyfus's (1991) advanced mathematical thinking suggests that careful instruction is needed for learning advanced mathematical concepts and for learning mathematical concepts that are not familiar from everyday life. Additionally, it suggests that mathematics is learned conceptually when students take charge of their own learning, when students are able to apply the same mathematical concepts in multiple contexts and applications, and when students are able to reflect on their learning of mathematical concepts.

2.3.4.3 Translations of a Mathematical Function

Lesh (1981) discusses that complete understanding of a mathematical concept requires the formation of three types of mathematical knowledge structures. First is "within-idea structures" which involve understanding the mathematical concept itself. Within-idea knowledge structures are coordinated systems of relations, operations, or transformations of a particular mathematical concept that distinguish it from other classes of the same concept. The meaning of a mathematical concept is usually context-dependent driven from its relationship with other concepts or from the system in which it is embedded. Therefore, second type of knowledge structure required to understand a mathematical concept is "between-concept structures" that include understanding the meaning of a particular mathematical concept in the context in which it is used. Third is "between-mode structures" that understand various representations of a mathematical concept together with organized systems of translation processes linking one representation to another. This includes "between-system mappings" (translations) and "within-system operations" (transformations).

Lesh's (1981) translation model proposes that a mathematical concept can be translated in five different representations, which are real-world translations, graphics, manipulateable models, words, and symbols. A complete understanding of any mathematical concept requires abilities to represent the concept in multiple representations and to make connections between the multiple representations. In a learner's mind, as the understanding of a concept develops, the related underlying transformation/translation networks become more and more complex. A mathematical concept is well learned when the learner can flexibly use a variety of relevant representational systems and easily switch to the most appropriate representation in a problem solving process.

Schoenfeld (1979) suggests that learning a specific mathematical concept in isolation leads to gap in recognizing problems where the concept can be used and identifying relationships between various concepts and strategies. Based on this, Lesh (1981) proposes that the fourth knowledge structure required to understand a mathematical concept would be organized systems of processes. A system of mathematical object may include more than sum of parts of systems of ideas (between concept structures), systems of operations and relations (e.g., within-concept structures), and systems of problem solving processes.

In a qualitative study conducted on fourth-grade students, it was reported that the ability to translate a mathematical concept significantly influences mathematical learning as well as problem-solving capabilities of students (Behr, M. J., Wachsmuth, I., Post, T. R., & Lesh, R., 1984). It was suggested that building the translational abilities facilitates conceptual learning and applying of mathematical concepts. A mathematical concept is well learned when a student can (1) recognize the concept embedded in a variety of qualitatively different representations, (2) flexibly manipulate the concept within given representations, and (3) accurately translate the concept from one system to another.

2.4 Three Studies on Problems in Learning Signals and Systems Course Content

In this section, I will present the results of three studies on the identification of students' difficulties in learning Signals and Systems course content.

2.4.1 Learning Difficulties and Knowledge Gaps in the Course Content and Potential Reasons behind Them

In the first part of this section (2.4.1.1), I will present the findings of a study that I conducted for my master's thesis (Fayyaz, 2009). In the second part of this section (2.4.1.2), I will suggest an explanation for these findings based on the learning theories discussed in section 2.3. This will illustrate some reasons for the choice of theoretical framework for this study.

2.4.1.1 Difficulties in Learning

This study was conducted for my master's thesis (Fayyaz, 2009). I started the study with the development of a detailed concept map of signal analysis through Fourier techniques, in order to identify difficult concepts. I started constructing the concept map by carefully adding all signal analysis related topics (identified through textbooks) and concepts (identified during classroom teaching of signal analysis related topics) required to conceptually learn signal analysis. The concept map was constructed after several iterations. Signal analysis content experts were consulted after each iteration. There were two purposes of the construction of the concept map, 1) Identify all the concepts and their linking prepositions required for conceptual understanding of signal analysis, 2) Identify difficult concepts by assessing the difficulty with which each concept was added or linked with other concepts in the concept map. The process of constructing and reflecting on the process of constructing the complete concept map revealed many difficult, misleading, and incompletely explained concepts in signal analysis. Later, I used this concept map to develop class tests and interview protocols to identify the concepts leading to misconceptions and learning hurdles for students taking Signals and Systems courses.

During this period, I was also teaching Signals and Systems, and Digital Signal Processing courses at a private engineering school in Lahore, Pakistan. I developed questionnaires and quizzes throughout the courses for three consecutive semesters to test students' understanding of the difficult concepts identified during the development of the concept map. The purpose of this exercise was to both verify the initially identified difficult concepts and test for additional learning hurdles. After each test or questionnaire, I conducted retrospective interviews with one or two students in each class to ask them about the concepts with which they struggled. Students' responses in class tests were used as a measure of difficulties in learning signal analysis, and interview responses were used to support the data collected in the class tests.

The problems identified in this study can be coded in seven categories: i) Difference between continuous and discrete domain, ii) discrete frequency, iii) units of Fourier series and Fourier transform, iv) periodic/aperiodic or finite/infinite duration signals?, v) sampling, vi) aliasing and folding, and vii) abstract mathematical concepts. The difficult concept discussed in one category may not be uniquely attributed to the problems in learning that particular concept, but rather a combination of more than one category of difficult concepts. The details of the problems identified within each category are given below.

2.4.1.1.1 Difference between Continuous and Discrete Domains

One of the major problems identified in learning signal analysis was the students' inability to differentiate clearly between a continuous domain signal and a discrete domain signal. These learning challenges might hinder successful understanding of various concepts like scaling, sampling, and transforms. These learning hurdles might arise from the following:

i. The same continuous x-axis is used to graphically represent both continuous and discrete domain signals as shown in Figure 2.1.

ii. There is no sophisticated and clear mathematical representation for signals when they convert from the continuous to the discrete domain. The currently used mathematical representation ($x(n) = x(t)|_{t=nTs}$) does not help much in understanding the drastic change from 't' to 'n' domain.

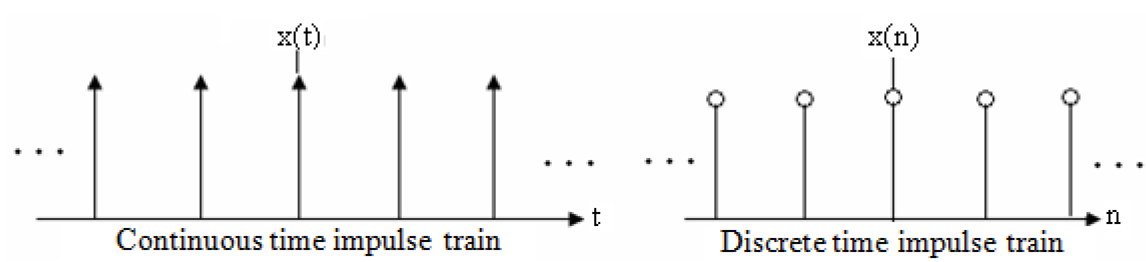

Figure 2.1. Same graphical representation of a continuous time signal and a discrete time signal on the x-axis (Fayyaz, 2009).

iii. A continuous time signal can be discrete in the frequency domain and a discrete time signal can be continuous in the frequency domain. This continuous interchange of the representations of the same signal keeps the student from distinguishing clearly between the two domains.

iv. A discrete domain signal is defined in textbooks as a signal that exists only for discrete values of the domain (e.g. time). However, instead of being undefined, a discrete domain signal is sometimes posited as having zero value in between the two discrete values of its domain. In this case, a discrete time signal is treated as a continuous time signal for representational purposes only. An example of such a case is upsampling of any discrete time signal as shown in Figure 2.2.

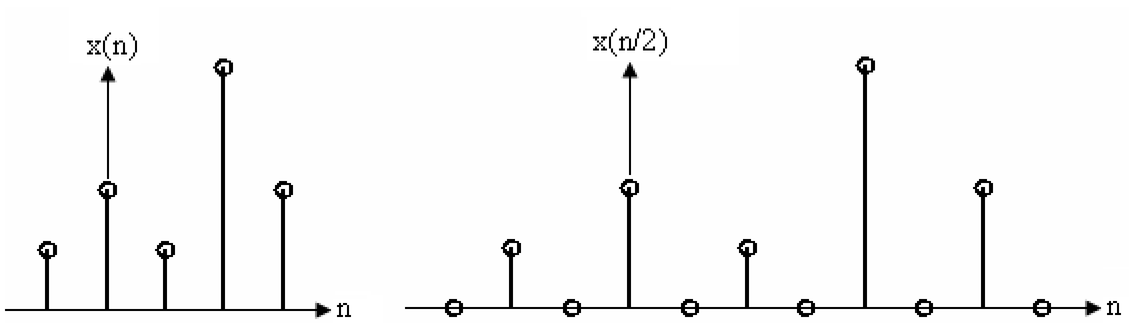

Figure 2.2. Upsampling a discrete time signal by a factor of 2 (Fayyaz, 2009).

v. There is a gap of information in textbooks on the process of transitioning from the continuous domain to the discrete domain (Figure 2.3). Additionally, no mathematical model exists for the conversion of sampled continuous time signals into a discrete time signal (Figure 2.4). This gap of knowledge in textbooks is a big stumbling block in learning signal analysis. Most teachers avoid acknowledging this discontinuity of knowledge in class, thus partially misleading students to think that a sampled continuous time signal is in fact a discrete time signal.

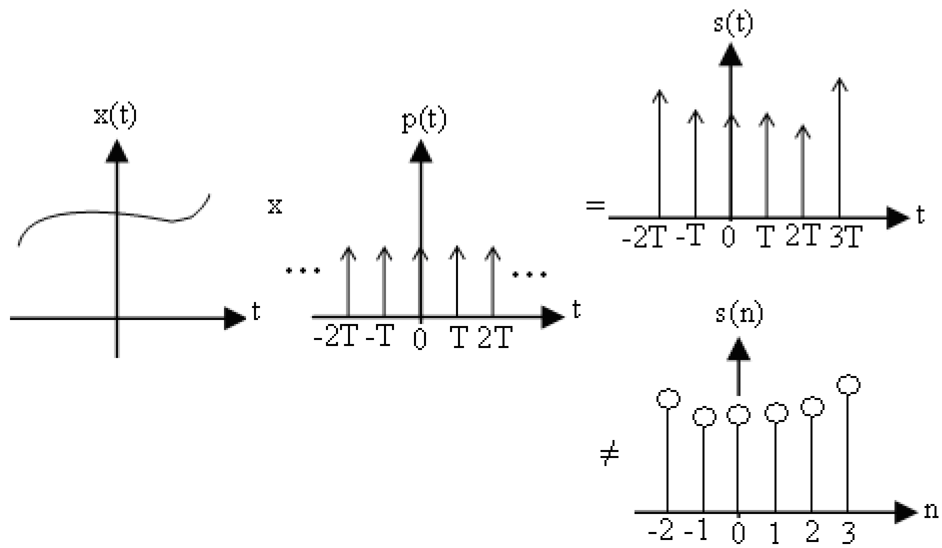

Figure 2.3. Multiplication of a continuous time signal with an ideal impulse train results in a continuous time signal and not a discrete time signal (Fayyaz, 2009).

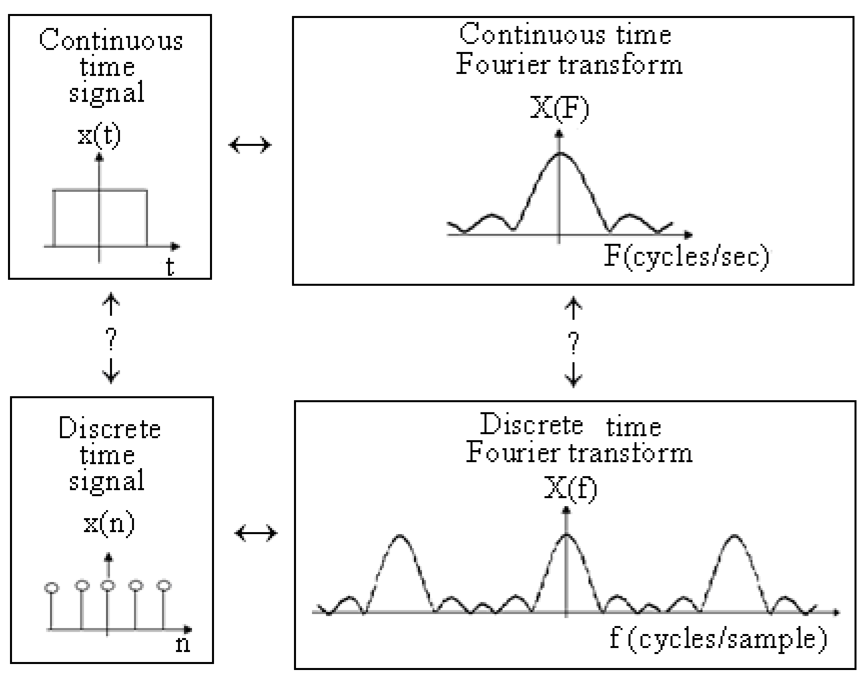

Figure 2.4. No sophisticated mathematical model explaining the conversion of a continuous time signal into a discrete time signal exists (Fayyaz, 2009).

2.4.1.1.2 Frequency of Discrete Time Signals

The conceptual understanding of the frequency of discrete time signals including the finiteness (modulo 2π) and periodicity of the frequency was another challenge found in learning discrete time signal analysis. This learning hurdle might emerge from the following:

i. The concept of a discrete time signal is introduced in Digital signal processing course by simply multiplying a continuous time signal with an impulse train. At this point, without a conceptual explanation, the quality of the frequencies of the resulting spectrum is changed from cycles per second to cycles per sample. This frequency (sometimes called as discrete frequency), f (units: cycles/sample) of the discrete time signals is explained in textbooks simply as the scaled version of the frequency, F (cycles/sec) of the continuous time signals ($f = F/F_S$, where F_S is the sampling frequency in samples/sec). None of the description and mathematical equations in textbooks helps to explain how the frequency of a discrete time signal becomes finite (modulo 2π). This creates confusion in connecting the concepts of finiteness and periodicity to the frequency of discrete time signals.

ii. A periodic signal is generally understood as a signal that repeats itself for all values of its domain. Therefore, the concept of periodicity yet finiteness of the frequency of discrete time signals create confusion in conceptually understanding the frequency of discrete time signals.

iii. The units of frequency of a discrete time signal is cycles/sample, which in general is written as Hz. This is the same as the generally used unit of continuous frequency, cycles/sec. This use of the same units for two different frequencies is confusing to understand the difference between the frequency spectra of discrete-time vs. continuous-time signals.

iv. The discrete frequency, in case of discrete time Fourier transform, is a continuous function of frequency. This mix-up of a discrete entity being continuous in domain inhibits the clear understanding of discrete frequency.

2.4.1.1.3 Units of Fourier Series and Fourier Transform

A clear understanding of the units of Fourier series and Fourier transform are important for conceptual understanding of different types of Fourier analysis, but are often not foregrounded in teaching material and classroom practices. One consequence of the lack of attention is the recognition of the difference between the quality of Fourier series and Fourier transform as stated in textbooks as a periodic signal can also have a Fourier transform, which is just a scaled version of its Fourier series. Whereas, in fact, Fourier transform is a function that is continuous within the frequency domain and Fourier series is a discrete within the frequency domain and representing a signal through Fourier transform instead of Fourier series is representing a signal that is continuous within its domain instead of discrete (Figure 2.5). The conceptual understanding of the existence of a difference in the nature of the spectrum demands the discussion of the difference in the units of Fourier series (e.g., Volts) and Fourier transform (e.g., Volts/Hz) as seen in Figure 2.5.

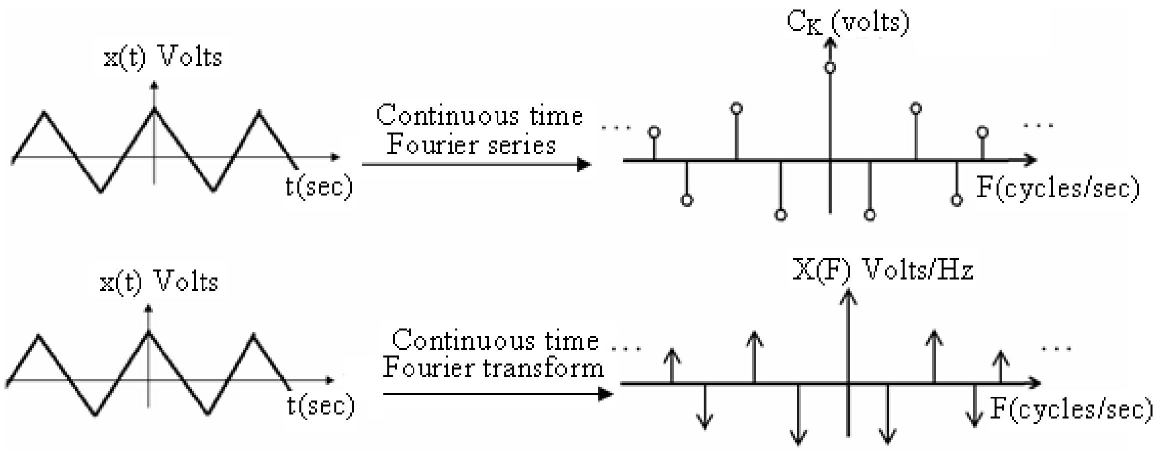

Figure 2.5. Graphical representation of Fourier series and Fourier transform of a periodic signal (Fayyaz, 2009).

2.4.1.1.4 Periodic/Aperiodic or Finite/Infinite Duration Signals

The concept of any transform (Fourier, s-, or z-, etc) is abstract, involves complex mathematical formulas, hard to understand intuitively, and disconnected from daily life. All these attributes suggest that conceptual learning of transforms can be a challenge. One of the challenges faced by students is the ability to decide upon the most suitable type of Fourier analysis technique for a particular signal. Some textbooks like Gray and Goodman (1995) describe periodic signals as finite duration signals and aperiodic signals as infinite duration signals. Gary and Goodman (1995) discusses Fourier series to be used to determine frequencies of a finite duration signal and Fourier transform for infinite, other textbooks (Oppenheim, Willsky, & Nawab, 1997; Lathi, 1998), however, attribute them to periodic and aperiodic signals. In addition, continuous time Fourier series and discrete time Fourier transform exhibit a confusing duality (Figure 2.6), which is never explicitly discussed in the textbooks. A lack of clear and standard categorizations of all possible signals might be a hurdle in learning.

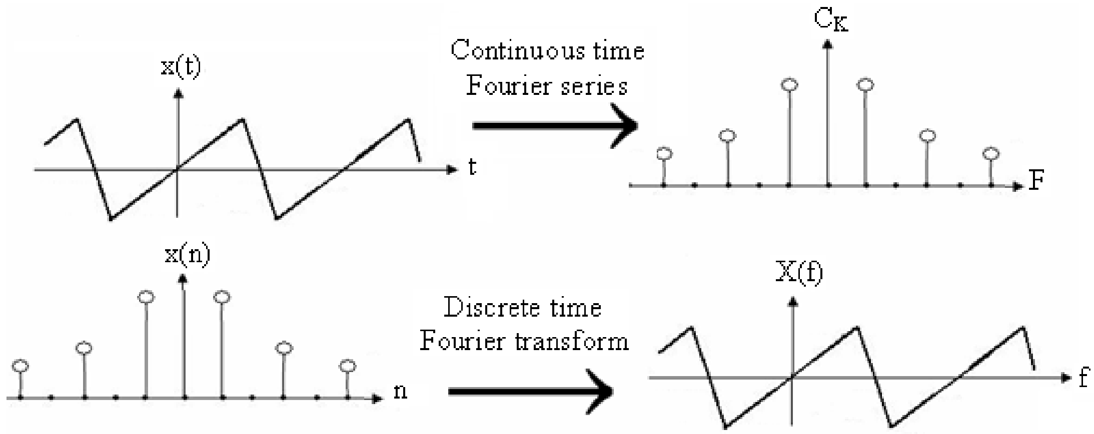

Figure 2.6. Continuous time Fourier series and discrete time Fourier transform appear as duals of each other (Fayyaz, 2009).

2.4.1.1.5 Sampling of a Continuous Time Signal

Two common methods of sampling are sample and hold and multiplication with a pulse train. Conceptual learning of the process of sampling is inhibited by the lack of adequate discussion of the role the sampling method plays in the results. For example, sampling a signal by either "sample and hold" or by "multiplication with a pulse train" results in two different signals (Figure 2.7). These representations cannot be proved equal either mathematically or theoretically in either time or frequency domain.

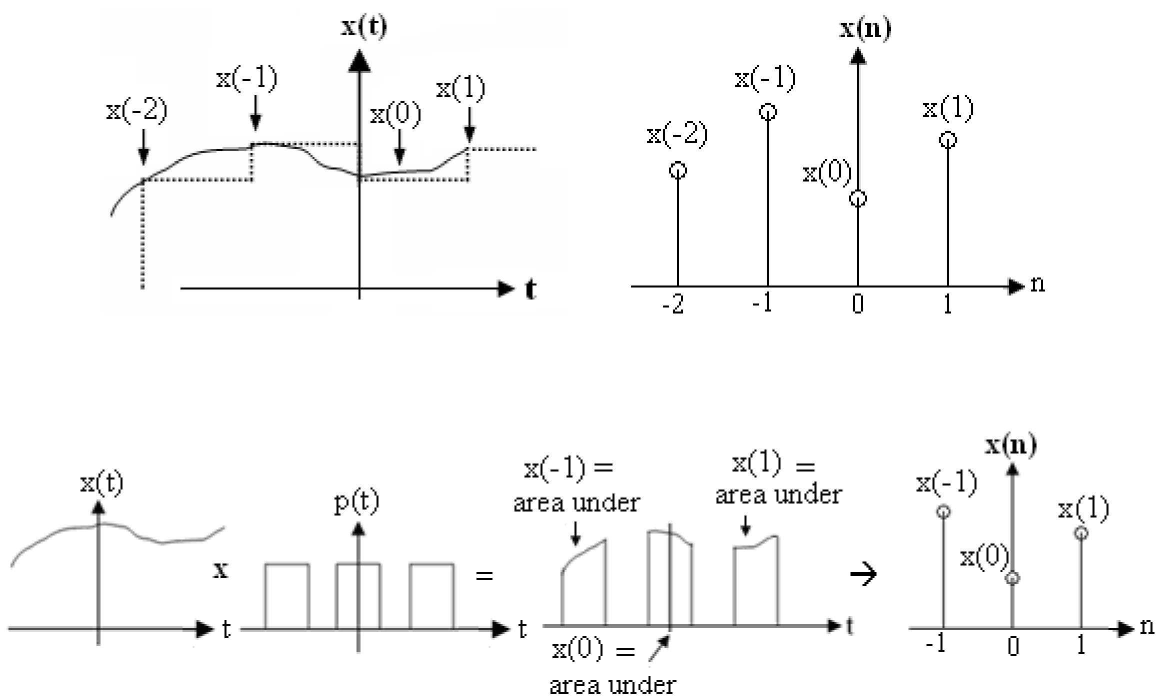

Figure 2.7. Discrete time signal obtained by sampling a continuous time signal by sample and hold followed by multiplication with a pulse train (Fayyaz, 2009).

2.4.1.1.6 Aliasing and Folding

When a signal is undersampled aliasing or folding occurs. Conceptualization of aliasing is difficult for students to intuit or even understand mathematically. One of the effects of aliasing is that the discrete time version of the undersampled signal may not resemble the original signal. As such, two different signals that appear to be very different in continuous time could result in the same discrete time signal if they are undersampled. It is very difficult for students to understand why and how this phenomenon occurs, much less to recognize it in both the time and frequency domains.

The concept of folding is another difficult concept for students. Students struggle because it involves understanding two difficult phenomena, phase inversion and aliasing, simultaneously. The concept of folding is easier to understand in the context of rotating objects like fans when the concept of phase can be related to the direction of rotation, but is harder to intuit in the context of angle of sinusoidal signals in the time domain.

2.4.1.1.7 Abstract Mathematical Concepts

While learning signal analysis, students continuously struggle with the intuitive as well as mathematical or scientific understanding of abstract mathematical concepts like s-, and z-domains, complex numbers, infinity, Dirac delta function, Kronecker delta function, complex exponentials, and Euler's identity. A few examples are:

i. A clear understanding of the difference between Dirac delta function and Kronecker delta function and more importantly, to decide which delta function should be used in a particular situation remains a difficult concept. The fact that these two deceptively similar looking delta functions also have deceptively similar looking transforms (Figure 2.8) further misleads students to treat both delta functions as discrete signals. The lack of a clear presentation of continuous and discrete domain signals as well as continuous and discrete frequency as already discussed above further manifests misconceptions in this situation.

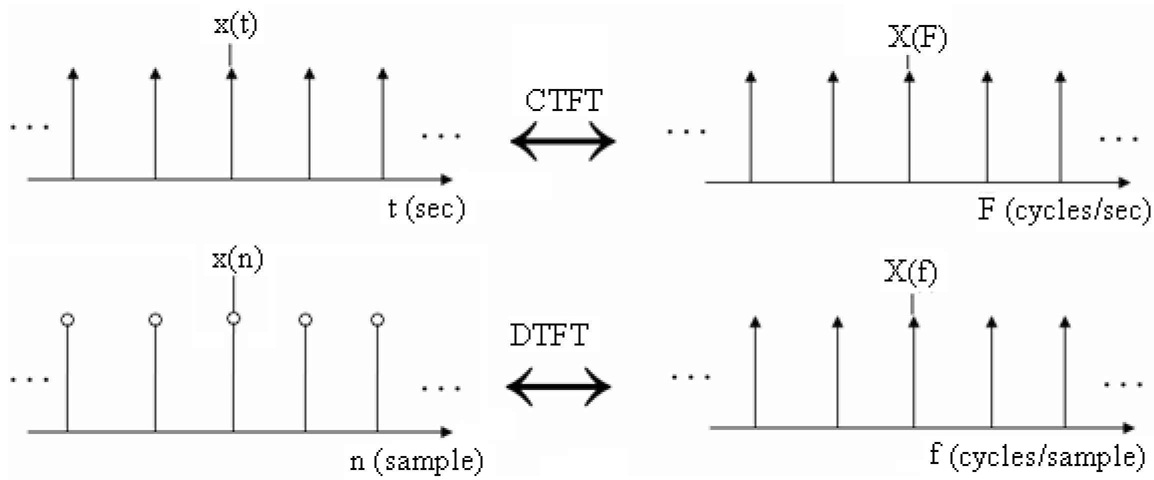

Figure 2.8. An ideal continuous time impulse train and a discrete time impulse train appear to have identical transforms (Fayyaz, 2009).

ii. Sinusoidal signals are introduced as complex phasors for the first time in this course. Conceptually connecting the concept of an oscillating signal with a vector rotating in the complex plane remains a challenge for the students.

iii. The spectrum of an impulse train is another impulse train (Figure 2.8). Additionally, the spectrum of the sum of infinite harmonically related sinusoids (each with zero phase shift) is also an impulse train. This means that the sum of infinite harmonically related sinusoids is an impulse train. Plotting the sum of hundred thousands of harmonically related sinusoids using MATLAB shows that the result is going close to being an impulse train as shown in Figure 2.9, but cannot be proved mathematically. Additionally, it is hard to understand intuitively.

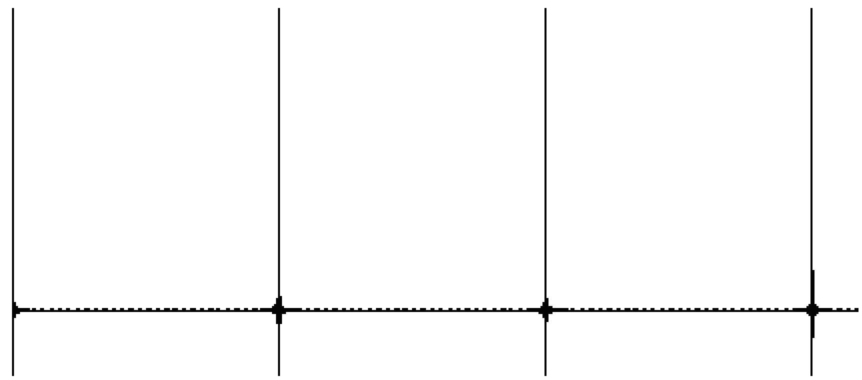

Figure 2.9. Plot of sum of hundred thousands of harmonically related sinusoids plotted on MATLAB (Fayyaz, 2009).

iv. The Fourier transform of an impulse train, an infinite sum of sinusoids, and an infinite sum of complex exponentials can all be represented as an impulse train, an infinite sum of sinusoids, or an infinite sum of complex exponentials. These facts are rarely presented explicitly in textbooks or in classrooms, which obstruct students' learning when they are encountered these interchanging representations.

v. Students face problems in differentiating between amplitude scale and time scale representations of signals, for example, 10x(t) and x(10t).

2.4.1.2 Explanation for Identified Misconceptions based on Conceptual Change Theories

In the previous section (section 2.4.1.1), various learning hurdles and difficult concepts in signal analysis were presented. This section discusses potential reasons behind these problems. I argue that all the difficulties discussed above can be attributed to category mistakes, p-prims, and/or excessive use of mathematical modeling and abstract concepts in signal analysis. I will first discuss the problems in learning due to category mistakes, then I will discuss problems in learning due to certain p-prims, and in the end of this section, I will discuss some learning hurdles due to excessive use of mathematical modeling and abstract concepts.

2.4.1.2.1 Category Mistakes

Some learning hurdles in signal analysis might be due to the obscure characterization of various ontological categories, like discrete and continuous domain, discrete and continuous frequency, taxonomy of signals, and real and complex signals. The description of these ontological categories and the misconceptions ascribed to them follows.

A continuous within its domain function and a discrete within its domain function are two mutually exclusive categories. However, the misrepresentation of concepts of signals within each domain (section 2.4.1.1.1) misleads students to confuse characteristics of continuous within their domain signals with discrete within their domain signals. This confusion is a category mistake. Explicit definition and categorization of the two types of signals might help to overcome many learning hurdles in signal analysis. For example, if students are clear about all the characteristics of the signals that are continuous within their domain and signals that are discrete within their domain, understanding the difference between Kronecker delta function and Dirac delta function will be a matter of mostly knowing their category membership.

Moreover, clearly categorizing the types of signals that are continuous within their domain and signals that are discrete within their domain might help students to overcome the hurdle of deciding about the appropriate Fourier analysis technique for a particular signal (section 2.4.1.1.4).

Additionally, the frequency of a continuous time signal and the frequency of a discrete time signal are two separate categories despite their seemingly similar characteristics (section 2.4.1.1.2). The clear distinction of the concepts and units within these two categories might help students to understand the different frequencies better. Furthermore, the proper understanding of the domain and the range of the frequency of continuous- and discrete-domain signals might help to rectify the common confusion that frequency of a discrete signal implies frequency, which is discrete within its domain (section 2.4.1.1.4). Additionally, the clear understanding of continuous- and discrete-domain signals and their classifications discussed earlier might support better understanding of the two frequencies.

Explicitly discussing the category membership of real and complex functions in this course might decrease confusion in understanding various functions. For example, sinusoidal signals are expressed as complex phasors in this course (section 2.4.1.1.7), which misleads students to consider sinusoidal signals as complex functions.

2.4.1.2.2 P-Prims

In addition to category mistakes, some of the learning hurdles in signal analysis could be due to the influence of naive knowledge structure, i.e., phenomenological primitives, of a student. A step towards improvement of students' understanding of signal analysis is the identification of the misconceptions that might be explained with the related p-prims and design instruction that can help to reveal and confront the appropriate p-prims. Some of the misconceptions that might be ascribed to the p-prims are discussed below.

Although the category mistake accounts for the tendency among students to treat a discrete time signal as a continuous time signal as discussed earlier, this bias could also be contributed from the p-prim concept of time in the minds of students. The p-prim that time is a continuous entity in everyday life may hinder the ability of students to understand various phenomena of discrete time signals, aliasing, and undersampling (sections 2.4.1.1.1 and 2.4.1.1.6). The inclination of students to treat the discrete domain as the continuous domain contradicts the framework theory claim that students tend to adhere to the concept of discreteness and natural numbers (Vosniadou & Verschaffel, 2004). I argue that this is because, for a novice, it is easier to think of discreteness in the context of counting and harder to think of discreteness in the context of time. This contradiction clearly supports DiSessa's (1988, 2008) claim of knowledge-in-pieces. In my opinion, the conceptual understanding of the discrete domain might improve if the students are able to replace the p-prim of continuous time from everyday life and confront the p-prim of counting in the context of time.

A few p-prims contribute to the hurdles in the conceptual understanding of the discrete frequency. One is the electric current as an example of a periodic signal, and the other is the concept of the unit Hz as cycles/sec. Before the introduction of the discrete frequency in this course, finiteness was never attributed to periodic signals. The definition of periodicity is not explicitly revised at this point either. Therefore, students may tend to connect the concept of periodic signals with the never-stopping sinusoidal electric signal they see in their daily lives. The activation of this p-prim in the context of discrete frequency might create a misconception in understanding of discrete frequency as a finite (modulo 2π) function (section 2.4.1.1.2). Additionally, although Hz is used as a unit for both continuous and discrete frequencies (section 2.4.1.1.4); students may associate the concept of Hz only with cycles per seconds. The p-prim that Hz is cycles/sec only creates hindrance in associating Hz as cycles/sample.

As discussed earlier, the learning of the discrete domain signals might get easier if students can confront their basic knowledge of natural numbers in the context of time. A few similar instances exist in learning signal analysis where students fail to appeal to their intuition (p-prim). Firstly, despite having prior understanding of the use of logarithms, which is also a type of transform, to solve certain kind of mathematical problems, students do not connect the concept of Fourier transform with logarithm. Secondly, the concept of complex phasors (section 2.4.1.1.7) might be better understood if students try not to connect it with the p-prim of counting, but rather connect it with the intuition of possible differences in the length and orientation of sticks in any given plane (Pickering, 2006).

2.4.1.2.3 Advanced Mathematical Thinking

Even if the students successfully understand different categories of signals (Figure 2.10), frequencies (Figure 2.11), domains of signals, and are able to acknowledge relevant p-prims, there are still threats to their conceptual understanding of some concepts in this course. These threats are due to the high cognitive demand of advanced mathematical thinking (section 2.4.1.1.7) and various incomplete and misleading instructional steps, mathematical models, and equations in several concepts in signal analysis. These concepts include graphical representations of discrete and continuous time and frequency axes (section 2.4.1.1.1), conversion of a sampled continuous time signal into a discrete time signal (sections 2.4.1.1.2 and 2.4.1.1.5), sampling (sections 2.4.1.1.1 and 2.4.1.1.5), definition and units of frequencies of continuous time and discrete time signals (sections 2.4.1.1.1, 2.4.1.1.2, and 2.4.1.1.3), relation between Fourier series and Fourier transform of a periodic signal (section 2.4.1.1.3), and abstract mathematical models (section 2.4.1.1.7). I argue that the incomplete mathematical models and equations are a result of more-than-necessary use of mathematics to simplify and explain otherwise complex processes in signal analysis. Bertrand Russell (1962) summarizes this phenomenon well when he states that physics is not mathematical because we know so much about the physical world, but because we know so little that we can only discover its mathematical properties. Furthermore, Dreyfus (1991) contends that mathematics is created through trial and error, partially correct and partially incorrect statements, and intuitive formulations with loose terms and imprecision, but is only taught in a polished form. This is why even when the students are able to perform standardized procedures using defined formulas, they lack the skill to flexibly use their knowledge of mathematics in unknown situations and even a slight change in the structure of a problem and/or formula can completely block their mental processes (Dreyfus, 1991). Based on this, I suggest that without addressing the gaps of knowledge that originate because of the use of mathematics alone to explain complex scholarship of 'Continuous-Time Signals and Systems' related topics, conceptual understanding of the subject is hard to achieve.

Additionally, in signal analysis, many concepts are abstract (section 2.4.1.1.7) which might make heavy cognitive demand from students. One example is a Dirac delta function. Even if students can characterize this function as a continuous function, its infinite height may continuously poses problems in learning this course content. The reason is that although abstract concepts are powerful in obtaining generalizations in mathematical concepts, they do not have very specific independent properties of their own. Abstract concepts are explained usually in context with their relationships with other similar or different concepts. Hence, understanding concepts that use abstract concepts in a certain context sometimes requires a mental ability to shift attention from the independent properties of the abstract concept itself to the structure of the properties and relationships of the abstract concept within the given context. Dreyfus (1991) suggests that learning an abstract concept gets more difficult because of the fact that it is difficult to make a visual image of an abstract concept in mind.

2.4.2 Difficult Questions in SSCI Post-Test

For the design of the questions for the protocol of this study, I analyzed continuous-time SSCI scores of undergraduate electrical engineering students at Iris University collected over ten years. The continuous-time SSCI v3.02 scores of 670 students and continuous-time SSCI v4.0 scores of 288 students were evaluated. It was found that majority of the students consistently answered a few questions and concepts incorrectly. The summary of the findings is shown in Table 2.1.

The table shows that the students face difficulty in understanding the concept that multiplication in the time domain corresponds to convolution in the frequency domain.

This conforms to Wage, Buck, Wright, and Welch's (2005) report on students' performance (25% students answered Q15 correctly) in SSCI v3.0. Additionally, students show a lack of understanding of the value of Fourier transform of a signal at zero frequency. Moreover, the scores show that students do not do well in questions on linear time-invariant (LTI) systems and causality. Convolution by graphical method is also another challenging concept. This conforms to Nasr's (2007) finding that students struggle with the concept of interval matching. Furthermore, students face difficulty in performing the combined operation of time shift and time flip on a signal.

Table 2.1. *Summary Analysis of Student Results on SSCI Tests*

Concept	Question Number and Percentage of Correct Answer	
	SSCI - v3 (sample size 670 students)	SSCI - v4 (sample size 288 students)
Multiplication in Time Domain	Q10 (2.8%), Q15 (34%)	Q11 (1.73%), Q22 (40.6%)
Fourier Transform and Area Under the Time Domain Signal	Q21 (44.7%)	-
LTI - Causality	Q23 (36.4%)	Q23 (46.875%)
LTI - Time Invariance	Q24 (20.2%)	Q24 (28.8%)
Convolution by Graphical Method	Q8 (52.53%), Q15 (34.02%)	Q13 (68%), Q15 (37.84%)
Time Flip and Shift Operations Combined	-	Q3 (64.5%)

2.4.3 Problems in Learning Signals and Systems Course Content across Borders

For my class project in *Qualitative Research Methods* course, I conducted a small study on the identification of problems in learning Signals and Systems course content across borders. For this study, I prepared questionnaires and sent them to a friend in Pakistan who was teaching this course at that time. Three undergraduate electrical engineering students in Pakistan wrote the answers to those questions and my friend scanned and emailed them to me. I conducted semi-structured interviews with three graduate students at a large research-intensive Midwestern university in the US where engineering has a strong presence, using the same questionnaire. I transcribed the interviews and compared the answers of all six students.

The results showed some similarities between the experiences of students across borders in learning Signals and Systems course content. All the participants reported that they were only able to learn the concepts superficially in this course. They attributed the reason for lack of conceptual understanding to the disconnection of the concepts from real life. Additionally, the participants revealed that even when they understood a certain concept; they were unable to place it in a bigger picture. One example of such a concept is convolution. They proposed that including more real life examples in this course might have helped them understand the course better. Furthermore, the participants mentioned that use of MATLAB was helpful in this course. The participants from both the countries said that this course was taught to them more like a mathematical course than an engineering course. Most of the participants from both the countries said that Fourier analysis is the most difficult topic in Signals and Systems course content whereas applying properties of Fourier transform is the easiest concept. My results are summarized in Table 2.2.

Table 2.2. *Summarized Results of Problems Encountered by Students in Learning Signals and Systems Course Content across Borders*

<table>
<tr><th></th><th>US</th><th>Pakistan</th></tr>
<tr><td rowspan="2">Easiest concept in this course</td><td>Laplace transform (1)</td><td>Properties of signals (1), operations on signals (1)</td></tr>
<tr><td colspan="2">Properties of Fourier transform</td></tr>
<tr><td rowspan="2">Hardest concept in this course</td><td>Translating between domains (1)</td><td>drawing of a spectrum (1)</td></tr>
<tr><td colspan="2">Fourier analysis, Convolution</td></tr>
<tr><td>How well you understood the concepts</td><td>Just accepted the way they are and did not try to rationalize them (2/3), Never did (1/3)</td><td>Only understood the mathematical parts</td></tr>
<tr><td>Use of computer tools (e.g. MATLAB) in learning this course</td><td colspan="2">Helpful</td></tr>
<tr><td>Ability to relate concepts learnt in this course with real life</td><td colspan="2">Not clear</td></tr>
<tr><td>Course treated more like a mathematical course or an engineering course</td><td colspan="2">Mathematical</td></tr>
<tr><td>While studying this course, ability to see where the concepts taught fit into the bigger picture</td><td colspan="2">No idea where concepts fit into bigger picture</td></tr>
<tr><td>Recommendations to improve this course to facilitate understanding the concepts</td><td colspan="2">More real life applications</td></tr>
<tr><td>Connection of convolution with everyday life</td><td colspan="2">No idea</td></tr>
</table>

2.5 Gaps in Research Conducted Thus Far in Learning Signals and Systems Courses

Nasr, Hall, and Garik (2007) identified a lack of in-depth qualitative research on students' understanding of Signals and Systems course content. They suggested that an effective pedagogical strategy could not be designed for this course without identifying the reasons behind students' faulty reasonings when they engage with Continuous Time Signals and Systems course content. To get a better understanding of students' faulty reasonings, they interviewed aerospace engineering students to identify faulty reasonings in concepts related to LTI electric circuits. However, Signals and Systems courses for electrical engineering students include additional important concepts, like, Fourier analysis, Laplace transform, etc. There is still a lack of 1) qualitative studies on the explanation for students' difficulties when they engage with the course content, 2) evidence on how students actually attempt to learn these concepts, and 3) understanding of cognitive resources that come into play when students access the course content for the first time. Additionally, there is inadequate research on how students learn the abstract concepts in this course.

Furthermore, there is a scarcity of research in learning of mathematical concepts related to Signals and Systems course content. The gaps in literature in conceptually learning concepts using extensive mathematical models and formulas include i) how mathematical formulas and modeling play a role in learning various engineering concepts, ii) whether learning theories explaining mathematical understanding also explain understanding engineering concepts through mathematics, and iii) how students engage with the abstract mathematical concepts like complex numbers.

This study investigates the above-mentioned gaps in the research in difficult concepts encountered by students while learning Signals and Systems course content. The findings of this study will facilitate the design of instructional strategies that help students to overcome their difficulties in learning the course content.

CHAPTER 3 - RESEARCH METHOD

3.1 Introduction

Learning conceptual knowledge in engineering science is crucial for developing competence and expertise in engineering (Streveler, Litzinger, Miller, & Steif, 2008). Signals and Systems is a core course in the undergraduate electrical and computer engineering curriculum (Wage & Buck, 2001; Nasr, Hall, & Garik, 2005). This course is difficult to learn conceptually because a significant number of topics in this course are abstract, are disconnected from a student's daily life, and make extensive use of the mathematical modeling and formulas (Nasr, Hall, & Garik, 2005; Ferri et al., 2009; Han, Zhang, & Qin, 2011; Tsakalis et al., 2011). The conceptual understanding of the content of this course is important, as these concepts become foundational knowledge for many other courses like communication, control systems, circuit design, image, and audio processing (Oppenheim, Willsky, & Nawab, 1997).

Despite many efforts to improve the learning experiences of Continuous Time Signals and Systems courses (Wage & Buck, 2001; Wage, Buck, & Wright, 2004; Cavicchi, 2005; Ferri et al., 2009; Han, Zhang, & Qin, 2011), the abstract nature of the concepts and their disconnection from daily lives have continued to pose difficulties in conceptual learning (Nasr, Hall, & Garik, 2007). This necessitates qualitative studies on how students engage with Continuous Time Signals and Systems course content (Nasr, Hall, & Garik, 2007) and what mistakes students make while trying to solve a related problem. The goal of this study is to fill the gap of qualitative research in understanding how undergraduate electrical engineering students engage with Continuous Time Signals and Systems course content by identifying their problematic reasonings. The research methods and rationale used to collect, analyze, and interpret data for this qualitative study is explained in this chapter.

3.2 Why Qualitative Research

Qualitative research is a method of inquiry that seeks to understand a social phenomenon within the context of the participants' perspectives and experiences. A qualitative study relies on the understanding of the views of the participants by typically asking broad and general questions to the participants (as opposed to specific and narrow questions in a quantitative study), and conducts inquiry in a subjective but unbiased manner (as opposed to objective in a quantitative study) (Creswell, 2002). Although literature may provide some information about the phenomenon of the study, a qualitative study focuses to learn from the participants through exploration (Creswell, 2002).

The purpose of this study is to understand the problematic reasonings employed by undergraduate electrical engineering students while learning and engaging with Continuous Time Signals and Systems course content. A qualitative research is most suitable for this study as the solution for the research problem requires both the exploration (how students conceptualize course content, what are the problematic reasonings employed by the students) as well as an understanding (because of its complexity) of the process of conceptually learning Signals and Systems course content.

3.3 Data Collection

Clinical interviews were conducted using semi-structured protocols for data collection for this study. Individual think-aloud interviews of the participants were audio-recorded. The details of the process of the protocol development, the protocol itself, process of conducting interviews, and various other choices made for this study are described in this section.

3.3.1 The Design Process of the Interview Protocol

This section will explain the process of the design of the protocol. The details of the protocol itself are presented in section 3.3.2. Eight questions were designed for this study and were divided into two protocols with four questions in each. The questions in the protocol were similar to textbooks or exam problems in typical Continuous Time Signals and Systems courses and were carefully designed to ensure that only very basic knowledge of Continuous Time Signals and Systems course content is required to answer each question correctly and completely. The questions in the protocol were related to signals and systems analysis in general, Fourier analysis, and convolution (details in section 3.3.2). There were a few reasons to use two protocols:

1. To increase the variety of the questions and topics for a one-hour interview

   A larger variety of questions and topics was preferred to increase the opportunity to identify conceptual problems that pervade in multiple contexts.

2. To increase the perceptual cues given to any participant

   In conceptual change interviews, asking a number of isomorphic problems that share the same solution strategy and conceptual knowledge but present different perceptual cues is recommended because such problems help to identify the degree to which students' access of conceptual knowledge is based on specific surface features of a problem (Streveler, Brown, Herman, & Montfort, 2014).

3. To ask different set of questions to the participants to reduce the possibility of a response guided by the sequence and types of the questions asked

   During conceptual change interviews, students often tend to solve a problem with the methodology they used in solving the most recent problem. This tendency to solve problems in a sequence with the same methodology can affect the revelation of true conceptual knowledge of the participant. This can be helped if participants are asked to solve a similar problem at a different time during the interview, because then they use different strategies and reasoning to solve the problem (Herman, Loui, Kaczmarczyk, & Zilles, 2012).

No rule was made about which participant will be interviewed under which protocol. The participants were interviewed based on their preferred available time and the two protocols were given to the participants alternately. Nineteen students participated in this study. Ten participants were interviewed under one protocol (Appendix C-Protocol B) and nine were interviewed under the other (Appendix C-Protocol A).

The questions in the protocol were designed based on:

1. The importance of a particular concept for students to know. Important concepts in the course were determined from consulting the literature (Chapter 2) and discussion with content experts (Table 3.5).
2. The degree to which undergraduate electrical engineering students understand a particular concept in Continuous-Time Signals and Systems course content. This was determined through i) literature, ii) analysis of ten-year long data of the Signals and Systems Concept Inventory scores of students at Iris University (section 2.4.2), and iii) discussion with content experts (section 3.10)
3. The strength of a question to obtain students' problematic reasonings related to the important concepts. The strength of each question was established through analysis of data collected from pilot interviews (sections 3.3.1.1, 3.3.1.2, and 3.3.1.3). To obtain rich verbal data, the questions in the protocol were carefully checked (through pilot interviews) to be neither too easy nor too difficult for students. Too difficult or too easy questions have the potential to hinder the ability of a student to talk through the process of problem solving as they can get stuck at difficult questions and may answer too easy questions too quickly without putting a lot of thinking in the problem-solving process (Sherin, 2001b).

Before the finalization of the protocol for the actual study, it was piloted, revised, and updated three times to ensure congruence between the research questions and each component of the proposed methodology. This was achieved by carefully analyzing multiple aspects of i) the interview settings (for example, resources that should be made available to the participants during the interview, time taken to answer each question, etc.), and ii) the quality of the data collected (for example, richness of verbal data collected for each question, clarity with which the participants understood each question, data provided to the participant for each question, etc.). The data collected from each pilot study was analyzed and the protocol was revised before the next pilot study. Existing literature, as well as experts in research in conceptual understanding in engineering education and/or 'Continuous-Time Signals and Systems' courses (section 3.10), were consulted at each developmental stage of the protocol to validate the changes made in the protocol. The details of each pilot study are presented next.

3.3.1.1 First Pilot Study

In the first stage of the pilot interviews, each protocol (Appendix A) was designed to last ninety minutes. During the interview, to ensure that the participants' responses were not affected by the concepts or formulas they could not recall, they were allowed to use textbooks or any resource (internet, etc.) with course related information. To accomplish this, I took with me a couple of standard textbooks and my laptop for the use of the participants during the interviews. Making these resources available was aligned with the goal of the study, which is to identify students' reasonings when they engage with the course content. Additionally, the participants were given questions on papers as worksheets and they worked on the solution on those worksheets with pen or pencil. These worksheets were collected back from the participants at the end of the interview and were later used together with interview transcripts for data analysis purposes. Six electrical engineering students from two large engineering schools voluntarily participated in the first pilot study. Of the six participants, two were graduate students and four were undergraduate students. The interviews were audio recorded and later transcribed verbatim. The data collected from the first stage of the pilot interviews was thoroughly analyzed to ensure that the questions in the protocol are well designed and aligned with the aim of the study. The protocol was revised (some questions were omitted and some were edited) after the analysis of the data obtained from the first stage of the pilot interviews. A few observations made after the first stage of the pilot interviews include:

i. The designed protocols were lengthy and each interview session took longer than ninety minutes. Additionally, it was observed that ninety minutes were too long to keep the participant interested and focused.

ii. The responses of the participants did not differ much before or after consultation from the textbooks or internet. Additionally, the time taken to respond to each question was drastically increased when participants used textbooks or any related material.

iii. Some questions were answered correctly by most of the participants so those questions were deemed ineffective for the purpose of the identification of problematic reasonings that lead to incorrect responses.

iv. It was a big challenge to make the participants think aloud. Thinking aloud is not a natural way of thinking for all and it was not natural for any of the participants. They either remained in the silent mode during the interview, forgetting that they were supposed to think aloud, or kept requesting to think quietly so they could think with concentration and report their thoughts later. Another challenge, in my opinion, ties to some of the learning difficulties already identified in learning mathematics related topics. Students generally access mathematics related content in a procedural manner rather than conceptual and they find it difficult to reflect on a procedure already well learned (now a habit).

v. In some instances, the participants used phrases like "this equation" or "that graph" and it was observed during data analysis later that audio data did not capture what the participants were referring to.

3.3.1.2 Second Pilot Study

Based on my experiences in the first pilot study, some changes (discussed throughout in this section) were made in the protocol (Appendix B), as well as in the interview settings for the second pilot study. These changes are described in this section.

For the design of the protocol, the questions that were deemed ineffective (either too easy or too hard) to gather knowledge of the participants' conceptual understanding in the first stage of the pilot interviews were omitted from the protocol. Moreover, the wordings of each question were adjusted according to the clarity that the participants demanded to answer each question in the first pilot.

For the second pilot study, the length of the interview was decreased to one hour to ensure active involvement of the participants throughout the interview. Additionally, I decided to not provide textbooks or internet access during the interviews after noticing in the first pilot study that the participants were i) taking too long to consult textbooks or internet during the interview, ii) not thinking aloud during the consultation, and iii) not responding very differently before or after consulting textbooks. However, a Fourier transform table and related formulas were still given to the participants so they can quickly access or recall the most commonly used Fourier transform pairs or formulas during the interview. The questions in the protocol were also verified to make sure that the response of each question only required basic conceptual knowledge of Continuous-Time Signals and Systems course content and not stipulate any memorization from the textbook. Additionally, although the protocol questions were designed in a way that they did not need any computation, calculator was given to the participants to save the time on calculating if they wanted to calculate anything.

In addition, the participants in the second stage of the pilot study were given questions on Microsoft Surface Pro instead of pages given to the participants in the first stage of the pilot interviews. The participants were asked to work on the tablet and click on the screen every time they had to talk about a particular graph or equation. The screen of the tablet was recorded using Camtasia Studio in addition to audio recording of the interviews to record richer details from the interviews.

In the second stage, the pilot interviews were conducted at Iris University (study site discussed in detail in section 3.4) to make sure that within the target population the designed protocol is well aligned with the aim of the study. Four undergraduate electrical engineering students participated in this pilot study. One major difficulty that continued to exist even in the second pilot study was to make the participants think out loud and provide rich verbal data.

3.3.1.3 Third Pilot Study

The challenge that continued in the second pilot study was that the participants were still not very comfortable with verbalizing their thoughts. Therefore, the questions in the interview protocol were further rephrased and revised for the third pilot study. These revisions were made based on the occurrences identified during the analysis of the data collected from the second pilot stage where the i) participants asked for further clarification or showed confusion, or ii) the researcher had to prompt the participants to provide additional details on their responses.

In the third stage, the protocol was piloted in an interview with one graduate student at a large Midwestern engineering school. A Fourier transform table, a formula sheet, and a calculator were provided to the participants as prior interviews proved this useful to the participants. This time the participant was not given the tablet and it was observed that when working with papers and pens the participant was more comfortable in verbalizing their thoughts. This observation prompted me to assume that one of the reasons for the lack of the verbal data collected from the second stage of the pilot interviews was limited writing space on the tablet's screen, which might have affected participants' comfort level of verbalizing their thoughts. Therefore, to capture detailed data in the actual study, I decided to video record the surface of the top of the desk where the participants worked instead of using the tablets to record the work surface. The need to use video data did not arise during analysis of the transcripts of the audio data collected for the actual study. Therefore, the video data recorded for the actual study was not used for the data analysis purposes.

3.3.2 Interview Protocol

Eight questions were designed for this study and were divided into two protocols with four questions in each (Appendix C). This section discusses the concepts covered in each question and objective behind the design of each question designed for this study. Table 3.1 summarizes the concepts covered in each question.

The first question was mainly centered on the concept of convolution. The participants were asked to i) perform the convolution of the two signals in the time domain, ii) perform the convolution of the two signals in the frequency domain, iii) find the relation between the result of convolution and the two given signals being convolved, and iv) find the relation between Fourier transforms of convolution result and the two given signals being convolved. The participants were given an option to use either graphs or equations only to perform the convolution. The two signals given in this question were an impulse train and a triangular function, which are very common signals in Continuous Time Signals and Systems courses. These two signals were specifically chosen to trigger instant recollection of the related concepts because of their high initial familiarity to the participants. Additionally, an impulse train was chosen because calculations with an impulse function do not take very long. This question was designed to i) understand how students perform the process of convolution, ii) explore how students understand the process of convolution for different domains, and iii) understand how students engage with an impulse train in particular and an impulse function in general.

The second question was about the relationship between the integral of an aperiodic signal and its corresponding periodic signal. This question was designed to understand how students conceptualize the graphical representation of the integral of a signal, and how students relate the properties (like area) of a periodic signal and its corresponding aperiodic signal.

The third question was centered on the concept of Fourier analysis of a signal. For this question, four different signals (an impulse function, a constant, a Fourier series, and a rectangular function) were given to the participants and they were asked to provide an explanation for the frequencies present in these four signals. The participants were expected to have a high initial familiarity, and hence quick recollection of related concepts and high confidence, with all four signals. Fourier transforms of these signals were also available in the Fourier transform table provided to the participants during the interview. Additionally, the participants were asked to explain the change in the frequency of an aperiodic signal after making the signal periodic. An aperiodic and a periodic rectangular function were given as examples for this question. The overall aim of the third question was to understand how students conceptualize the mathematical equations given in the Fourier transform table and obtained from the Fourier analysis procedure in the context of the frequency components of a signal.

The fourth question was centered on the concept of linearity and time-invariance of systems. Participants were asked to describe their understanding about time-invariance and linearity of a system and use their knowledge to find time-invariance and linearity of the two given systems. The information about one system was given in the form of the mathematical equations and the information about the second system was given in the form of the graphs. This question was designed to explore how students understand the concepts of linearity and time-invariance of a system and apply their knowledge to different systems and representations. Due to an oversight on my part, one part of this question (Q4.b) had incomplete information. I gave the mathematical equation for the impulse response of the system instead of input and output signals. Later during the discussion with one of the content experts (section 3.10), we realized that this is incomplete information to determine linearity and time-invariance of the system. The analysis method specifically for this question is discussed in section 3.7 in detail.

The fifth question (Q1 in Protocol B, Appendix C) was centered on the concepts of Fourier series and Fourier transform. In this question, the participants were given signals in the form of mathematical equations. All the signals were represented as sinusoids or exponentials or combination of sinusoids or exponentials. The purpose of this question was to see how students understand frequencies of signals that are already in the form of sinusoids or exponentials.

The sixth question (Q2 in Protocol B, Appendix C) was centered on the concepts of time shift, time scale, and phase shift. Participants were given two signals expressed in mathematical equations and they were asked to scale and shift those signals together in time. Of the two signals, one was a phase shifted sinusoidal signal. The purpose of this question was to understand students' i) reasonings behind performing the combined operations of time shift and time scale on a signal and ii) perception of the difference between phase shift and time shift of a signal.

The seventh question (Q3 in Protocol B, Appendix C) was centered on the concept of Fourier transform and the conditions for the existence of Fourier transform. In this question, students were given a signal that did not satisfy the sufficient conditions of the existence of Fourier transform and some altered forms of the same signal that satisfied the condition. The purpose of this question was to understand students' reasonings as they find the Fourier transforms of the signals.

The eighth question (Q4 in Protocol B, Appendix C) was focused on the general concepts of Fourier analysis and convolution. The purpose of this question was to explore participants' understandings of the concepts of convolution and Fourier analysis apart from their mathematical formulas.

Table 3.1. *Summary of Concepts Covered in Each Interview Question*

Question #	Concept(s) Covered
1	Convolution
2	Area under a periodic signal and corresponding aperiodic signal
3	Fourier transform
4	Linear and Time-Invariant Systems
5	Fourier series and Fourier transform
6	Time scale, time shift and phase shift
7	Fourier transform and sufficient condition for existence of Fourier transform
8	Fourier Transform and Convolution

3.3.3 Clinical Interviews

In this section I will discuss i) the reasons for using interviews for collecting data for this study, ii) the type of interviews used for this study, iii) steps taken to reduce the effects of the inherent disadvantages of using interviews as a data collection tool, and iv) interview settings for this study.

3.3.3.1 Why Clinical Interviews

A few major categories of qualitative data collection are observations, interviews, documents, and audiovisual materials. A qualitative interview is chosen when a researcher wants to ask open-ended questions to the participants to get their voice on their experiences unconstrained by any bias of the researcher or the existing literature. Additionally, interviews give some flexibility to the researcher to control the type of information received in the interview, which helps to collect the data that cannot be directly observed (Creswell, 2002). Clinical interviews help to explore an individual's unique mental processes and expose hidden structures in an individual's thinking (Seidman, 1998). Throughout the history of conceptual change research, clinical interviews are considered most consistent with the assumptions of cognitive constructivism (Ginsburg, 1997) and are used extensively as the primary means of accessing students' understandings (McDermott & Shaffer, 1992). The goal of this study is to uncover the reasonings employed by students in their responses that lead to difficulties in solving problems related to Continuous Time Signals and Systems courses and clinical interviews are the most appropriate method to flexibly achieve this goal (Nasr, 2007).

3.3.3.2 Type of Clinical Interview Chosen

One of the derivatives of the clinical interview method is verbal reporting. The theoretical basis of this method lies in the cognitive model of information processing (Ericsson & Simon, 1980). Verbal reporting method is classified into two main categories based on the temporal relation of verbal reporting and cognition. These are concurrent (think-aloud) and retrospective. Concurrent reports are the ones in which the participants verbalize their thoughts while performing a task (Ericsson & Simon, 1980). Verbal reporting while solving a task particularly facilitates researchers to investigate thinking processes of the participants while solving mathematics related tasks that otherwise only generate right and wrong answers (Ginsburg, Jacobs, & Lopez, 1998). Retrospective reports are the ones in which the participants verbalize their thinking after completion of the task (Ericsson & Simon, 1984). Some studies also discuss a third type of verbal protocols called predictive reports. In predictive reports, the participants verbally talk about their potential performance before performing the task (Ginsburg, Jacobs, & Lopez, 1998).

This study employed clinical interviews exploiting both concurrent and retrospective verbal reporting techniques. At first, the participants were asked questions related to the topics taught in Continuous-Time Signals and Systems courses and they talked out aloud while concurrently trying to solve the given problems. Once the participants had solved a particular problem, if needed, the interviewee asked them questions in retrospect for any clarification or further understanding of their responses, for example, among all possible ways to answer a particular question why the participant chose the method he/she used to solve the given problem, etc.

3.3.3.3 Disadvantages of Clinical Interviews and Measures Taken to Minimize Them

Collecting data through interviews has disadvantages too. I took some measures to take care of the disadvantages of the interviewing technique used. The disadvantages and measures taken to minimize their effects include:

i. Disadvantage: The data collected by the interviews is a participant's perspective only (Creswell, 2002).

To alleviate this problem, this study makes sure to capture data that represents the conceptual difficulties among a broad population. This is achieved by using a relatively larger sample size and piloting the protocol thrice. In the three pilot studies, I thoroughly analyzed the data collected at the end of each study before the next to ensure that the identified problems in learning are representative of a larger population. Additionally, discussions with content experts (experts in conceptual learning as well as experts in Continuous-Time Signals and Systems course content described in section 3.10) were done at each stage for the validity of the protocol used for this study.

ii. Disadvantage: Interviews only measure participants' conscious and stated knowledge of their cognitive processes (Garner, 1987).

To alleviate the effect of this disadvantage, the data collected from the pilot interviews for this study was carefully reviewed to identify the questions and concepts where the participants' responses were not very clear or the participants were unable to give a detailed response. The questions in the protocol and their language were constantly revised and rephrased in the three pilot stages of the development of the protocol so that the questions sound familiar to the participants and they appropriately understand the intent of the question.

iii. Disadvantage: Interviews include the possibility of the influence of the presence of the interviewer on the response of the participant (Orne, 1969; Adair, 1984; Ericsson & Simon, 1984; Shaochun, Zendi, & Xuhui, 2007).

To make the participants comfortable with their responses during the interview, they were told in the beginning of the interviews that they will not be judged based on any right or wrong answer and the aim of the study is just to understand their thought processes. In addition, the participants were ensured before the interview that any faculty in their university will never know about how they responded during the interview and their identities will be protected. Furthermore, no faculty member was present at the time of the interview to ensure participants' comfort in the interview settings.

iv. Disadvantage: Data collected by the interviews necessitates a very accurate interpretation (Davison, Robins, & Johnson, 1983; Jaspers, 2009).

To ensure correct interpretation of the data collected for this study, the researcher's interpretation was validated by multiple discussions with the content experts (discussed in section 3.10) so that the researcher's bias is minimized and multiple lenses are incorporated to look at the data.

v. Disadvantage: In the analysis of the data collected through interviews, the researcher has to be very careful about correct interpretation of the silences in the data (Boren & Ramey, 2000; Benbunan-Fich, 2001).

This study mainly looked at the reasonings used by the participants when they talked through the process of solving the questions given to them during the interviews and silences were not considered as data for this study.

vi. Disadvantage: Interview necessitates proper recording and careful transcription (Creswell, 2002).

To ensure noise-free recording, the interviews were conducted in a quiet room on campus. This also provided uninterrupted attention of the participants. Additionally, this study used good quality audio and video recorders, and used the services of professional transcribers for transcription of the audio data. The audio-recorded data was transcribed verbatim for data analysis purposes. Furthermore, the work surface of the participants' desk was video recorded to make sure that all the responses were captured completely (video data, however, was not used for this study because the need for additional information did not come up in the recorded audio data).

vii. Disadvantage: The abundance or dearth of language data collected from interviews can be the greatest asset or liability of interviews as data collection methodology (Pressley & Afflerbach, 1995).

Pilot studies for this study were specially focused to ensure that the interview questions are phrased very clearly and prompted rich language data. Additionally, the participants were provided with calculator, a Fourier transform table, and related formulas so that the cognitive load on them is reduced and their responses are not constricted by the need to have additional resources to answer a particular question.

3.3.3.4 Interview Settings

The interviews were conducted in a quiet office at Iris University (study site described in section 3.4). Only the researcher and the participant were present in the office at the time of interview. The main interview questions were the questions from the pre-designed protocol (Appendix C); however, the discussions were kept flexible to follow up on the response of the participant, if needed.

IRB approval was not initially sought for the first pilot study because publishing the data was not the original intent to conduct the pilot study. I wanted to use those interviews just as practice to hone my interviewing skills and improve the protocol for the actual study. The participants were explicitly told about the initial intent of the interviews. The interviews were successfully completed and useful data was collected from the interviews. The richness of the data prompted a need to present the findings to the engineering education community, so decision was made to apply for retroactive IRB approval. For this purpose, I contacted the participants again and explained the situation to them and asked if they were willing to sign the consent form (Appendix E) given IRB approves the study. All six participants agreed and after that, I applied for the retroactive IRB approval, which was approved (Appendix D). All the interviews for the pilot studies later and the actual study were conducted after the IRB approval. (Appendix F). All the participants of the subsequent pilot studies and the actual study signed an informed consent form (Appendix G) before the start of the interview. The consent form had details of: i) the purpose of the study, ii) the expectations from the participants; iii) the expected length (in time) of one interview; iv) potential benefits of participation, v) incentives to participate; vi) securing confidentiality of the participants' names and their data; and vi) the rights of the participants to opt out of the study. Additionally, the participants were offered the summary of the results when the study is completed. Two participants showed interest and a summary will be sent to them once the results are published. To encourage participation and to compensate the participants for their time and support (Wolcott, 2002), each participant was given a $20 (as per IRB's recommended amount) check request form redeemable from the Business Office of Iris University.

3.3.3.5 Length (in time) of One Interview Session

The interview protocol for the actual study was designed to take no longer than one hour. However, there is always a chance a discussion can take longer than the allocated time. At the beginning of the interviews, participants were informed that if the discussion went past the allocated time, it would be at their discretion to stay. A one-hour long interview was considered reasonable to make sure that a participant does not get bored or tired towards the end of the interview. This was decided based on my observation from the ninety-minute pilot interviews that the keenness of the participants reduced after an hour of participation. The time taken by the participants during pilot interviews to answer each question was carefully monitored to design a protocol that was expected to finish in an hour. The minimum time taken by any participant in the actual study was 32 minutes and the maximum time taken by any participant was 1 hour 43 minutes. The median of time taken by all the nineteen participants of the actual study was 1 hour 9 minutes.

3.4 Study Site

For qualitative research, participants and sites are chosen to best achieve the central phenomenon of the study. For this reason, standard practice is purposeful sampling of the participants as well as the site (Creswell, 2002). This study was conducted in a teaching-intensive Midwestern university, which, to preserve anonymity, will be referred to as Iris University. Iris University has been among the list of top US undergraduate engineering colleges for the many consecutive years now. In an annual survey conducted by U.S. News and World Report for its 2013 college guidebook, Iris University was highly rated by America's engineering deans and senior faculty members, and was ranked on top for five engineering areas including electrical engineering. The minimum requirement to apply for the admission in any program in Iris University is that the applicants must be in the top 25% of their high school graduating class. Based on the above-mentioned facts about Iris University, it was considered a very suitable site for this study because it can be assumed that any random volunteer participant from this university has a good academic background and their problems in learning Continuous Time Signals and Systems course content are not influenced by their poor academic backgrounds. This fact strengthens the practical implications of the results obtained from this study, as the problematic reasonings identified in this study are representative of the problematic thinking processes of high-achieving students.

3.5 Description of Continuous Time Signals and Systems Course Taught at Iris University

At Iris University, Continuous Time Signals and Systems is a four credit-hour course, taught in a 10-week quarter system, which includes three 50-minute lectures and one 160-minute laboratory session per week. The topics covered in Continuous Time Signals and Systems course at Iris University include continuous time signal modeling, Fourier series and Fourier transforms, response of systems to periodic and aperiodic continuous time signals, filter design, and sampling. The course typically ends with sampling theory (course outline is presented in Appendix J). The lab content consists of various hands-on application-oriented activities that provide students with personal experiences with Continuous Time Signals and Systems course content. These include simultaneously observing their ECG signal in the time domain and its spectrum in the frequency domain. The pre-requisite courses for Continuous Time Signals and Systems course at Iris University include one year of physics (three quarters with four 50-minute lectures and one 3-hour lab per week for each quarter) and one year of Calculus (three quarters with five 50-minute lectures per week for each quarter). This means that at Iris University students get 6000 minutes of physics and 7500 minutes of calculus instruction as a pre-requisite. In schools with the semester system, students typically take two semesters (sixteen weeks with three lecture hours or 5760 minutes of instruction per semester) each of calculus and physics (data provided by a faculty member at Iris University). Additionally, two courses of differential equations and two courses of AC and DC circuit analysis are pre-requisite courses for Continuous Time Signals and Systems at Iris University. Students at Iris University typically take Continuous Time Signals and Systems course in the Fall or Winter quarter of the third year of undergraduate electrical engineering program while all math and physics courses are typically completed by the Winter quarter of the second year.

3.6 Participants

This study focuses on the undergraduate electrical engineering students' problematic reasonings when they engage with Continuous Time Signals and Systems course content. The target population for this study was undergraduate electrical engineering students who have already taken Continuous Time Signals and Systems course. I intended to identify problematic reasonings that pervade the conceptions of high-achievers, so I limited the target population to students who have passed the Continuous Time Signals and Systems course. This course, at Iris University, is offered to electrical engineering undergraduate students in the start of junior year, so the target population of this study was undergraduate electrical engineering students who were either in their junior year or higher. This section discusses the selection of sampling method and the sample size for this study.

3.6.1 Sampling Strategy

Participants for this study were selected using stratified purposeful sampling strategy. Stratified purposeful sampling lays out the characteristics of particular subgroups of interest and facilitates comparisons between subgroups. Stratified sampling can be used only when enough information about the sample population is known to classify the samples. Stratified purposeful sampling extends credibility to a research study (Patton, 2001).

From the target population, this study aimed to recruit two different groups of students based on their academic statuses. One group, called the CTSS-only group, comprised of students who have only taken one Continuous Time Signals and Systems course and no subsequent courses. The second group, called the CTSS-plus group, consisted of students who have also taken one or more courses that require prior knowledge of Continuous Time Signals and Systems course content. The courses at Iris University that require prior knowledge of Continuous-Time Signals and Systems course content are Communication Systems, Linear Control Systems, Electromagnetic Waves, and Discrete-Time Signals and Systems. This criterion of sample selection and division of participants in groups of different academic levels was intended to identify the difference in the problematic reasonings employed by students with different levels of knowledge and exposure to applications of Continuous Time Signals and Systems course content. Additionally, this criterion helps to identify robust problematic reasonings that persist even when a student continues to apply these concepts in related courses and applications (Montfort, Brown, & Pollock, 2009).

3.6.2 Sample Size

Qualitative research has no standard rules for sample size. Various factors like research questions, purpose of the inquiry, credibility criteria, available time, and resources collectively contribute to determine the appropriate sample size (Patton, 2001). Even the sample size of one, if selected purposefully can be sufficient in some cases for in-depth analysis (Patton, 2001; Creswell, 2002). Mason (2010) suggests that the "sample size in the majority of qualitative studies should generally follow the concept of saturation when the collection of new data does not shed any further light on the issue under investigation" (p. 8). He examined 560 doctoral theses to determine the appropriate sample size for qualitative interview-based research and reported a mean sample size of 31 interviews and standard deviation of 19 interviews. However, heterogeneity of the population might require a larger sample size (Mason, 2010).

The participants of this study are undergraduate electrical engineering students from the same university (homogeneous population). There was no gender preference for the selection of the participants for this study for two reasons. First, Iris University is a small school and restricting the participation based on the gender could have posed difficulties in finding the desired number of participants. Secondly, the scope of this study does not account for gender specific learning differences.

I decided to recruit twenty students for this study based on the i) general trends in the sample size for similar qualitative studies, ii) number of questions I wanted to ask each participant during the interview, and iii) length of one interview session. Nineteen students volunteered to participate in the study. Therefore, the sample size of this study became nineteen.

3.6.3 Recruitment Process

The participants were recruited to participate in this study through:

1. A "call for participation" email sent from secretary of the Department of Electrical and Computer Engineering at Iris University to all the junior and senior students in the Department of Electrical and Computer Engineering at Iris University (Appendix H).
2. Class announcements made by the researcher in a few classes (digital systems, discrete-time signals and systems, principles of design, and software defined radio). These classes were chosen over other classes in which the enrolled students were expected to have passed Continuous Time Signals and Systems course because these classes were meeting on the day the researcher planned to make the announcements (recruitment Script is in Appendix I).

3. Requesting electrical engineering faculty at Iris University to encourage students in their classes to participate in this study

The students were told to directly contact the researcher for consent to participate. The faculty at Iris University had no knowledge of the participants of this study.

3.6.4 Sample Profile

Of the nineteen students who participated in this study, eight (CTSS-only group) had taken only one Continuous Time Signals and Systems course and no subsequent courses, and eleven (CTSS-plus group) had taken one or more (up to four) such courses. The subsequent courses taken and passed by the participants at the time of participation are shown in Table 3.2. Pseudonyms are assigned to the participants to protect their privacy.

Table 3.2. *Courses Passed by Participants which Require Prior Knowledge of Continuous-Time Signals and Systems Course Content*

Number of Participants	Pseudonyms	Courses Taken	Group Label Used in the Study
8	Emily, Mark, Erin, John, Matt, Kevin, Jim, Jake	None	CTSS-only
1	Ryan	Discrete-Time Signals and Systems	CTSS-plus
2	Megan, Lily	Communication Systems, and Discrete-Time Signals and Systems	
1	Tom	Linear Control Systems, Electromagnetic Waves, and Discrete-Time Signals and Systems	
3	Luke, Rick, Caleb	Communication Systems, Electromagnetic Waves, and Discrete-Time Signals and Systems	
4	Carl, Paul, Justin, Bill	Communication Systems, Linear Control Systems, Electromagnetic Waves, and Discrete-Time Signals and Systems	

Additionally, the gender and nationality of the participants of this study are given in Tables 3.3 and 3.4 respectively. The knowledge of the gender and nationality of the participants help to understand the variety of the participants of this study. At the time of the study, in the Department of Electrical and Computer Engineering at Iris University, 14.2% of the undergraduate students were female and 17.3% of the undergraduate students were international. This sample is representative of the overall population in the Department of Electrical and Computer Engineering at Iris University.

Table 3.3. *Genders of Participants*

Number of Participants	Gender
16	Male
3	Female

Table 3.4. *Nationalities of Participants*

Number of Participants	Nationality
16	US National
3	International

3.7 Data Analysis

In qualitative data analysis, the researcher makes a personal assessment to describe what best fits the situation or themes that capture the major categories of information. The researcher's interpretation may differ from someone else's interpretation. This does not imply that there is any right or wrong interpretation, but only means that every researcher brings his or her own perspective in interpreting the data (Creswell, 2002). Thematic analysis (Braun & Clarke, 2006) was used to analyze the data for this study. This section discusses how data for this study was analyzed. The final codebook is provided in Appendix K.

1. The Interview sessions were audio recorded and transcribed verbatim for data analysis purposes. The researcher transcribed seven pilot interviews for this study. Professional transcribers transcribed four pilot interviews and nineteen interviews for

the actual study. The transcripts were reviewed against the recorded audio to check for accuracy. Moreover, the work surface of the participant's desk was video recorded to ensure all responses are captured completely. However, the video data was not analyzed for this study.

2. Before the start of the data analysis, solutions of the interview questions were made by the researcher and double-checked by consultation with a faculty member at Iris University who has been teaching Continuous Time Signals and Systems courses for eight years. Most of the questions in the protocol could be solved by a variety of approaches and the questions were phrased in a way that the participants had flexibility to choose any approach (intuitive, mathematical equations, graphs, etc.) to answer a particular question. The researcher and the faculty member also discussed and agreed upon multiple possible approaches to solve each question.
3. The transcriptions were thoroughly read and all the problematic reasonings, mistakes, and demonstrations of missing conceptual knowledge were separately collected in an excel sheet. For this study,
    a. A reasoning is defined as the participant's purposeful effort to generate justifiable conclusions and make sense of the problem.
    b. A problematic reasoning is defined as a reasoning that has the potential to hinder conceptual understanding and cultivate misconceptions. For example, participants' thought processes behind their choices for their actions, spoken aloud by them, while incorrectly solving a mathematical equation.
    c. A mistake is defined as the incorrect response of the participant without enough evidence of the reasoning employed behind it. For example, a

participant's incorrect attempt to solve a mathematical equation without explaining the thought processes behind the steps taken.

d. Missing conceptual knowledge is defined as the knowledge that was not evident in the participants' responses and the use of which could have helped the participant to successfully solve the problem at hand. This was identified by the instances where the participants were asked to explain something and they either just said that they didn't know (different from when they said they did not know because they did not remember or could not recall), or they also explained what they did not know, or they were stuck at some question and kept brainstorming but could not figure out the certain type of knowledge that could have helped them to get out of that stuck situation.

4. Keeping the context together with the problematic reasonings, mistakes, and demonstrations of missing conceptual knowledge collected from the data helped in multiple iterations of coding and recoding of the data (Braun & Clarke, 2006). It also facilitated in understanding the variety of possible topic areas in Continuous Time Signals and Systems course content where conceptual learning could be influenced by one particular problematic reasoning (section 4.2), mistake (section 4.4.1), or missing conceptual knowledge (section 4.4.2).

5. Since the focus of this study is to identify problematic reasonings, a response was not considered as a problematic reasoning, a mistake, or the demonstration of some missing conceptual knowledge if

   a. The participants self-corrected themselves either in the same question or later during the interview in any other question or context.

  b. The participant did not answer a particular question at all for reasons such as "I do not remember how we did this in the class," etc. because this study focuses only on the problematic reasonings and not on measuring the retention of Continuous Time Signals and Systems course content.

6. A response or statement was considered a problematic reasoning, a mistake, or the demonstration of some missing conceptual knowledge if the participants made an incorrect statement in the given context and never self-corrected themselves in the same question or later during the interview in any other question or context. In addition, one question in the protocol (Q4.2) had incomplete information due to oversight on my behalf (details discussed in section 3.3.2). The response of the participants for that question was analyzed based on whether the participants were able to identify that some data was missing in the question or not.
7. Once all the problematic reasonings, mistakes, and occurrences of missing conceptual knowledge from all the interviews were collected together, similar statements either made by the same participant in different questions or made by different participants were grouped together to understand them collectively. This grouping also helped to identify if a particular problematic reasoning originated from one question (context) or more. This information helped to identify prevalent problematic reasonings.
8. At this stage, different content experts (section 3.10) were consulted to establish the difference between mistakes and problematic reasonings identified and to identify common thinking patterns of the participants.
9. In the first round of coding, the incorrect statements were grouped under the name of the topic associated with them. For example, an incorrect statement that originated in

a question related to Fourier series was categorized under Fourier analysis. I started exploring thoughts of the participants associated with each incorrect statement in the second stage and moved the incorrect statements around the thoughts or actions of the participants behind each statements. For example, if a particular incorrect statement was about wrong graphical translation of a signal, it was placed under the category of graphical challenges. Subsequent cycles of coding were about further exploring the thoughts and incorrect statements and narrowing down the focus to get the actual problematic reasoning behind the undesired responses (codebook is given in Appendix-K).

10. A problematic reasoning, mistake, or missing conceptual knowledge was attributed to the participants' preference to use mathematical equations over other options if the participant failed to answer the question correctly because of relying on just mathematical formulas when clearly an additional (and at some instances only) use of intuitive or graphical approach would have been helpful.
11. Discussions were made with content analysis experts in all stages from second stage onwards to validate the understanding of participants' reasonings from the evidence collected from the data.

3.8 Quality of the Research

Qualitative researchers use different terms to describe research quality like accuracy, credibility (Creswell, 20012), authenticity, trustworthiness (Lincoln & Guba, 1985), etc., and use different strategies and terms to validate their choices and decisions, like member checking, auditing (Creswell, 2002), etc. I will mainly use the Qualifying

Qualitative Research Quality (Q3) typology presented by Walther, Sochacka, and Kellam (2013) for interpretive research in engineering education to explain how I established the quality of this research. This typology includes theoretical, procedural, communicative, pragmatic validation, and process reliability.

As per Q3 typology, validity of an engineering education research can be described as the "agreement of the results of the measured quantity" (Sirohi & Krishna, 1983, p. 39), "extent to which the research findings appropriately reflect properties of the social setting investigated" (Walther, Sochacka, & Kellam, 2013, p. 636), and "process character of the framework" (Walther, Sochaka, & Kellam, 2013, p. 637). Additionally, the reliability of an engineering education research can be described as "the repeatability of a measuring process" (Sirohi & Krishna, 1983, p. 40), "an attempt to mitigate the effect of random influences on the research process" (Walther, Sochaka, & Kellam, 2013, p. 637), and "an accessible way of conceptualizing research quality without assuming the positivist stance of the intellectual traditions from which these terms originated" (Walther, Sochaka, & Kellam, 2013, p. 637).

Theoretical validation focuses on whether the "concepts and relationships of the theory appropriately correspond to the social reality under investigation?" (Walther, Sochaka, & Kellam, 2013, p. 640). For theoretical validation of this study, stratified purposeful sampling strategy was used for participant selection. Additionally, the sample for this study is chosen to best represent the research topic and to ensure effective and efficient saturation of categories, with optimal quality data. Furthermore, methodological coherence (Morse, Barrett, Mayan, Olson, & Spiers, 2002) was ensured in this study by matching the research question with appropriate method of data collection (interviews), and matching data collection method with the most appropriate data analysis method (thematic analysis) for this study. Content experts were consulted to ensure methodological coherence and sample appropriateness for this study.

Procedural validation focuses on "which features of the research design improve the fit between reality and the theory generated?" (Walther, Sochaka, & Kellam, 2013, p. 640). For procedural validation of this study, content experts were consulted for protocol development, and data analysis. Additionally, to reduce the risk of the misinterpretation of the data, analysis was done in iterations. Content experts (to be discussed in section 3.10) were consulted at each iteration and the process is documented in the form of dated copies of revised versions of the data analysis document. Additionally, the researcher made sure to bracket her bias with piloting the process of developing the protocol and conducting the interviews. Furthermore, researcher's biases were checked by continuous discussion with content experts at each stage of protocol development, data collection, and data analysis.

Communicative validation focuses on whether "the knowledge is socially constructed within the relevant communication community?" (Walther, Sochaka, & Kellam, 2013, p. 640). For communicative validation of this study, the method and findings at different stages are presented in four different workshops (Simoni, Aburdene, & Fayyaz, 2013b, 2013c, 2013d, 2014) over a period of one year. In these workshops, we opened discussions on protocol development, methods of data collection and data analysis for this study with the participants of the workshops. These workshop presentations provided external auditing for this study. Furthermore, publications are planned from the findings of this study to further open this work for discussion among related communities. In addition, interactions with experts (section 3.10) at all stages of the study contributed to communicative validation too.

Pragmatic validation focuses on whether "concepts and knowledge claims withstand exposure to the reality investigated?" (Walther, Sochaka, & Kellam, 2013, p. 640). For pragmatic validation of this study, the protocol was piloted in three developmental stages and with students in three different engineering universities. Additionally, we intend to present our research results to a variety of communities including educators, education researchers, engineering educators, and engineering education researchers, and electrical engineering students. The extent to which our results resonate with people in these different communities will validate pragmatic validity of this study.

Process reliability focuses on "how can the research process be made as independent as possible from random influences?" (Walther, Sochaka, & Kellam, 2013, p. 640). For process reliability of this study, the protocol was piloted in three developmental stages and with students in three different engineering universities to ensure that the data obtained from the protocol is as independent of random factors as possible. The data after each pilot stage was thoroughly analyzed to improve reliability of the protocol, interviewing skills of the researcher, and the process of data analysis before the subsequent stage. Content experts were consulted at each pilot stage. The pilot studies and the actual study were IRB approved and adhered to the standard methods for data collection, data coding, and data analysis.

3.9 Reducing Researcher's Biases

Qualitative researchers do not typically use the word "bias," as qualitative research is interpretive. However, it is important for the researchers to reflect on their role in the study based on their personal experiences, interests, and values that shape various steps of the research study (Creswell, 2002). Following are my prior experiences with the learning and teaching of this course, and the details of the steps taken throughout the study to reduce the biased influences of the prior experiences on my decisions.

I have taught Signals and Systems courses numerous times in Pakistan and have a fair understanding of the concepts covered in this course, which might pose a challenge to appreciate, understand, and interpret the problems faced by the students when they attempt to learn this course. Furthermore, before I started this study, based on my prior teaching experience, I had some pre-notions about the common mistakes students make when they access the course's content. For this reason, I remained very careful in designing and conducting the interviews not to let any personal bias influence the design of the interview protocol or discussions during the interviews. Moreover, I was careful to hide my excitement or disappointment on receiving any expected responses during the interviews. Additionally, while understanding participants' conceptual knowledge and thinking processes, I stayed very mindful to detach my identity as a teacher of this course and stay in the role of a qualitative researcher.

To maintain complete awareness of my decisions and reduce the influence of my biases throughout the study, in addition to constant reflection, I consulted experts in engineering and/or engineering education (section 3.10) during all the stages of the study, which include developing the protocol, conducting the interviews, and analyzing the data.

Furthermore, the design of this study and preliminary results at different stages were discussed in various workshops and presentations to invite discussions and suggestions from content experts.

3.10 Description of External Content Experts

To establish the validity of this study, experts from the fields of qualitative research, STEM education, and/or Signal Processing were consulted at various stages. The summary of the content experts' profiles and areas in which they assisted the study is given in Table 3.5.

While there is no standard rule for sample size in qualitative research, sample size decisions generally follows the concept of saturation (Mason, 2010). I followed the same concept of saturation to determine the appropriate number of content experts for this study. I consulted several experts who helped shed light on the problem under investigation at each stage of this study. After consulting with ten experts comprised of seven experts in Continuous Time Signals and Systems course content and seven experts in qualitative research methods, it became clear that I had reached the stage of satisfactory communicative and procedural validation and consulting with more experts would not add any more insight to the coding of the data. I then discontinued further consultation.

Primarily, for this study, a specific person was considered an expert in a particular field if that person had extensive teaching and research experience in that particular area.

Three senior faculty members (1, 2, and 3 in Table 3.5) with (teaching and research) expertise in Signal Processing were consulted for protocol development, data analysis, and inter-rater reliability for this study. Furthermore, an expert in mathematics education research (4 in Table 3.5) was consulted at the data analysis stage to discuss the themes identified in the data that involved mathematical thinking. Additionally, to establish the quality of the qualitative methods employed in this study, two faculty members (5 and 6 in Table 3.5) with expertise in qualitative research in engineering education were consulted at various stages of protocol development, data collection, and data analysis.

Secondarily, I also consulted the experts who were recent graduates and students in the PhD program in electrical engineering or engineering education who had a master's degree in electrical engineering as well as teaching experience in electrical engineering related subjects (7, 8, 9, and 10 in Table 3.5). I considered them experts because of the combination (knowledge of signal analysis and knowledge of qualitative research in engineering education) of their academic and research training.

Table 3.5. *Credentials of Experts Consulted and the Areas in which they Assisted*

	Current position	Research Experience (in years)	Teaching Experience (in years)	Area(s) in which they Assisted
1	Faculty member at Iris University	Active learning and conceptual understanding of students in Continuous Time Signals and Systems courses (10)	a. Electrical engineering courses in general (13) b. Continuous Time Signals and Systems courses (8)	a. Protocol development (course content) b. Data analysis (course content)
2	Faculty member in the School of Electrical and Computer Engineering at a large Midwestern research-intensive university	Electrical and computer engineering undergraduate content curriculum area (13+)	Electrical engineering related courses (8)	a. Protocol development (course content, qualitative research methods) b. Data analysis (course content, qualitative research methods)
3	Faculty member in the Electrical and Computer Engineering Department at a private teaching-intensive Mid-Atlantic liberal arts college	Electrical engineering related courses including linear systems & signal processing (30+)	Electrical engineering related courses including linear systems and signal processing (30+)	a. Protocol development (course content) b. Data analysis (course content)
4	Faculty member in the Department of Curriculum and Instruction at a large Midwestern research-intensive university	Mathematics education (12)	a. Mathematics education (12) b. Mathematics (10)	Data analysis (mathematics related content and qualitative research methods)
5	Faculty member in the School of Engineering Education at a large Midwestern research-intensive university	Difficult concepts within science and engineering (20)	Engineering education related courses (20)	a. Protocol development (qualitative research methods) b. Data analysis (qualitative research methods)
6	Faculty member in the School of Engineering Education at a large Midwestern research-intensive university	a. General research (14) b. Engineering education research (14) c. Engineering design (14) d. Math and mathematical thinking in engineering design (10) e. How children learn engineering concepts from their parents (7)	a. General (9) b. Engineering and engineering education related courses (7)	a. Protocol development (qualitative research methods) b. Data analysis (qualitative research methods)
7	Faculty member in the Department of Engineering and Physics in a small Southern private liberal arts university	Engineering education (4)	Continuous Time Signals and Systems courses (4)	Data analysis (course content, and qualitative research methods)
8	PhD candidate in the School of Engineering Education at a large research-intensive Midwestern university with graduate degree in Electrical Engineering	Engineering Education (2)	High School (4)	a. Protocol development (course content, and qualitative research methods) b. Data analysis (course content, and qualitative research methods)
9	PhD candidate (Mechanical Engineering) at a large private research-intensive Western university	a. Student and alumni career pathways b. engineering skill and identity development c. curricular improvement	a. Undergraduate level statics b. Graduate-level courses: finite element analysis, graduate design research methods, graduate engineering education assessment methods	Protocol development (qualitative research methods)
10	PhD candidate (Electrical Engineering) at a large private research-intensive Western university	Computer architecture and VLSI design	Graduate-level courses (Very-Large-Scale-Integration design and circuits, computer architecture, graphics hardware)	Protocol development (course content)

CHAPTER 4 - RESULTS

4.1 Introduction

This chapter is divided into three sections. The first section presents the findings of this study that answer the first research question, "What problematic reasonings do undergraduate electrical engineering students employ when they engage with Continuous Time Signals and Systems course content?" As mentioned earlier (sections 1.6, and 3.7), for this study, a *reasoning* is defined as a person's purposeful effort to generate justifiable conclusions and make sense of the problem, and a problematic reasoning is defined as a reasoning that has the potential to hinder conceptual understanding and cultivate misconceptions (definitions created specifically for the study). Moreover, a misconception is defined as any aspect of an individual's conceptual beliefs and frameworks that resists conceptual change and contributes to incorrect, naive, or unproductive conceptual understanding (Streveler, Brown, Herman, & Montfort, 2014).

The second section of this chapter presents the differences in problematic reasonings between students who have taken only one Continuous Time Signals and Systems course and no other course that require prior knowledge of the course content and students who have taken such subsequent courses. This will answer the second research question of this study; "How do these problematic reasonings differ after the students take more courses that require prior knowledge of Continuous Time Signals and Systems course content?"

The third section of this chapter presents some additional findings from student responses that go beyond strictly answering the two research questions. Although the aim of this study was to identify students' problematic reasonings, there were some noteworthy instances in the data where the participants gave incorrect, inappropriate, or incomplete responses and were not very clear in describing their thoughts behind their responses. The prevalent themes of such instances were deemed important to mention for conceptual learning and effective teaching of this course. For this study, these findings are labeled as mistakes and missing conceptual knowledge. A mistake is defined as the incorrect response of the participant without enough evidence of the reasoning employed behind it (definition created for the study). In addition, missing conceptual knowledge is defined as the knowledge that was not evident in the participants' responses but the use of which could have helped the participant to successfully solve the problem at hand (definition created for the study). This chapter concludes with the summary of all the findings presented throughout the chapter.

To protect the privacy of the participants, pseudonyms are used in the presentation of the data throughout this chapter. As mentioned in Chapter 3 (section 3.6.1), participants are grouped according to the courses they had taken at the time of the study. These groups are the CTSS-only (students who have taken no subsequent courses after Continuous Time Signals and Systems course) and the CTSS-plus (students who have taken and passed subsequent courses). Table 3.2 can be referred to for the details of the academic profiles of the participants.

4.2 Problematic Reasonings

As discussed in section 2.1.1, content of a typical Continuous Time Signals and Systems course can be categorized into three main content areas:

1. Signal representations and operations (SRO): Includes topics like mathematical and graphical representation of signals, components of signal (even, odd, etc.), types of signals, various operations on signals like time shifting, time scaling, etc., complex signals like Dirac delta, sinc, unit step function, etc.
2. Frequency Analysis (FA): Includes analysis of signals through Fourier series and transform.
3. System Analysis (SA): Includes topics like discussion of types of systems with emphasis on linear time-invariant systems, impulse response, and LTI system analysis through convolution and Laplace transform

The collected data from the conducted interviews suggests the following problematic reasonings (in no particular order) of the undergraduate electrical engineering students when engaging with the Continuous Time Signals and Systems course content. Explanations for these problematic reasonings will be discussed later in this section. These problematic reasonings are further categorized (in no specific order) according to their related content areas, shown in Table 4.1.

1. Any property of a signal is limited within the duration of the signal itself.
2. $\delta(t)$ and $\delta(\omega)$ are functions like $x(t)$ which varies according to whatever value t takes on.
3. The product of any function and an impulse function is a constant.
4. A periodic signal in the time domain is also periodic in the frequency domain.

5. Signal representation in the time domain is same representation in the frequency domain.
6. A constant in the frequency domain means no frequency as it has no ω in it.
7. Phase shift means shifting the phase plot of a signal in the frequency domain
8. Convolution and multiplication are interchangeable.
9. Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance.

Table 4.1. *Problematic Reasonings Employed in Continuous Time Signals and Systems Course Content*

Signal Representations and Operations (SRO)	Frequency Analysis (FA)	System Analysis (SA)
SRO1. Any property of a signal is limited within the duration of the signal itself. SRO2. δ(t) and δ(ω) are functions like x(t) which varies according to whatever value t takes on. SRO3. The product of any function and an impulse function is a constant.	FA1. A periodic signal in the time domain is also periodic in the frequency domain. FA2. Signal representation in the time domain is same representation in the frequency domain. FA3. A constant in the frequency domain means no frequency as it has no ω in it. FA4. Phase shift means shifting the phase plot of a signal in the frequency domain	SA1. Convolution and multiplication are interchangeable. SA2. Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance.

This study was not designed to equally explore the problematic reasonings related to each content area (Signal representations and operations, frequency Analysis, system analysis). Therefore, the difference in the number of the problematic reasonings in each category does not imply any comparison of a content area being more difficult to learn than the others are. The details of all the problematic reasonings as per their content area and the data suggesting the presence of these problematic reasonings is presented below.

4.2.1 Signal Representations and Operations (SRO)

The explanation of the problematic reasonings related to 'Signal Representations and operations' content area and the examples from the data collected for this study suggesting the presence of these problematic reasonings are presented in this section.

4.2.1.1 Any Property of a Signal is Limited within the Duration of the Signal Itself (SRO1)

The data recommends that the students think that any property of a signal is limited within the duration of the signal. A few examples from the data supporting this reasoning are presented below:

The participants were asked to find and draw the integral $s_1(t)$ of a rectangular function $v_1(t)$ shown in Figure 4.1. The correct limits on x-axis for area under the integral of the rectangular function are from $t = -1$ to ∞. Irrespective of the different responses given by the participants, they chose the same time limits on the x-axes of the plots of the signal $v_1(t)$ and its integral $s_1(t)$. Some responses of the participants are as follows:

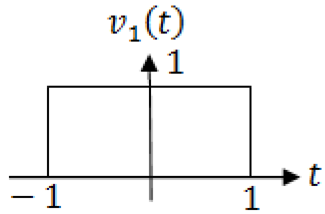

Figure 4.1. A rectangular function.

John found the total area under the rectangular function to be 2 and said, "I would do it, just do the area. So, it's the length of 2 and then height of 1 and it would just be equal to 2" and started drawing the plot of the integral. While drawing he said,

> The integral is just feels like is you're building up. It would just steadily increase until you got to the end. That's why I feel like, well, so it's thinking in my mind right now why it would look like that, so ... because just the integral taken from this value going up and you're just compounding on it, so it just would be a steady increase and then just stop there ... I feel like it would be ... this point here would be 2. (John)

Although John correctly identified and drew the shape of the area under the rectangular function for time between -1 and 1 as shown in Figure 4.2, he terminated the plot of the integral at t=1, just where the rectangular signal $v_1(t)$ ended.

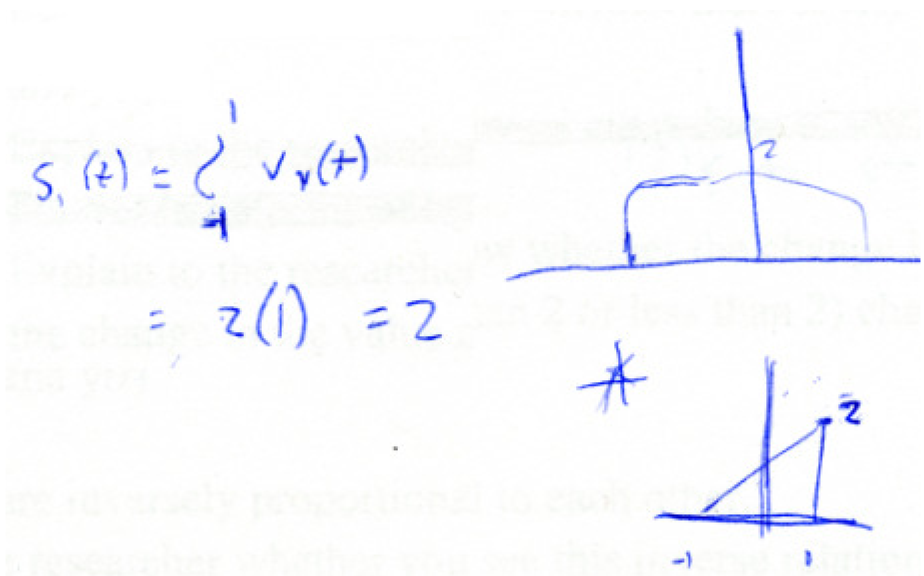

Figure 4.2. John's response for the area under the rectangular function.

Additionally, Emily (Figure 4.3), Tom (Figure 4.4), and Justin (Figure 4.5) drew similar plots for the area and limited their plots within the time limits of the actual rectangular function. Their responses are as follows:

> So I have my rectangle function, and it goes from negative one to one. Amplitude of one. So my integral since it's a signal would go from negative infinity to infinity, but because it's a rectangle function it's only there from negative one to one. And then it would be rect(t/2), and then centered about one d(t), but since it's only on it would be negative one to one. One d(t) and then t integrated from negative one to one, so that would be two, would be my answer, and then I guess I would just plot a straight line across that two. (Emily)

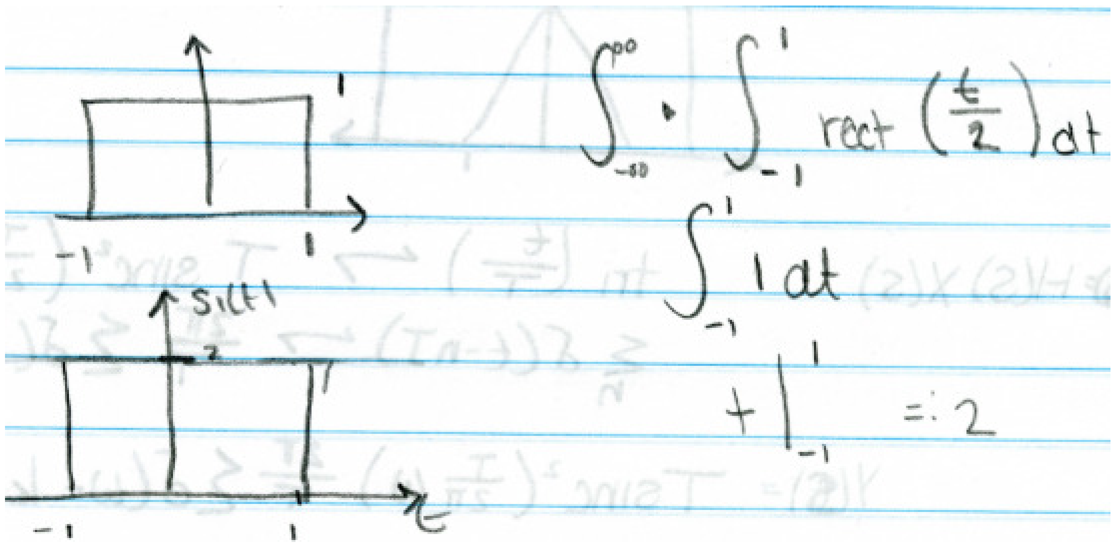

Figure 4.3. Emily's response for the area under the rectangular function.

Let's see. Area beneath the curve from negative 1 to 1 ... . So, at first glance, I would say it's constant ... . Hmm. So, my thoughts on this would be, if I were going to take the integral of this function from negative 1 to 1, it's just 1 from those during that time, then I would end up with maybe 1 to 1 of the $v_1(t)$, which is just 1 dt. I'm going to get t that goes from evaluated at negative 1 to 1. And so, if I take this and say 1 minus negative 1, I'm going to get 2. And ... I guess it would just be that then, negative 1 to 1, 2, because it's constant. This was constant 1 during that range, and if I integrate that, I'll end up getting a larger value. That's how I would, that's what I would say. (Tom)

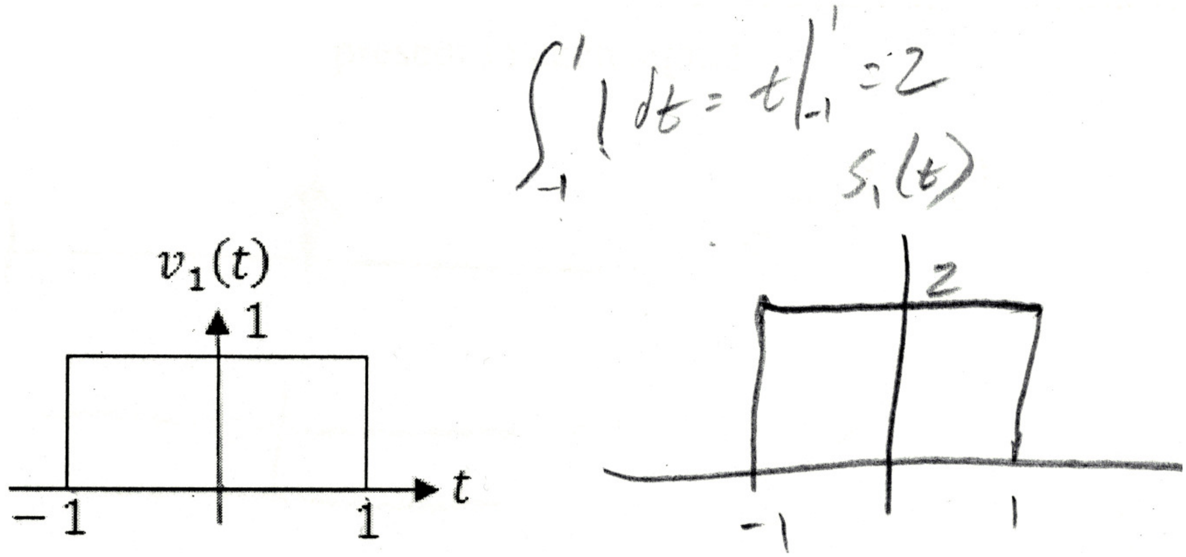

Figure 4.4. Tom's response for the area under the rectangular function.

So I would integrate $v_1(t)$ from negative 1 to 1. And that would be my $s_1(t)$, which is pretty much when you integrate, it's pretty much just the area underneath the shape that you have over here ... . So we know that this is your ... so this is from negative 1 to 1, so we know that the length here is 2. The length of signal's 2, and the amplitude is 1. This is your $v_1(t)$. And then your $s_1(t)$ is equal to the integral of $v_1(t)$. And we know the limits are from negative 1 to 1, so we take the area underneath here. I would say that your $s_1(t)$ would be 2. And then … I would just draw that as a constant. (Justin)

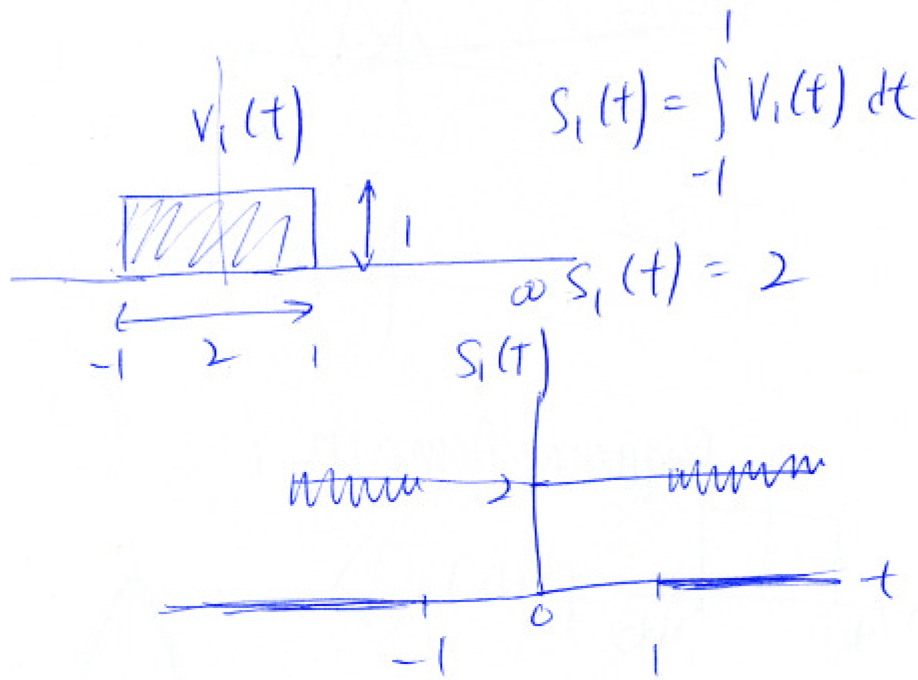

Figure 4.5. Justin's response for area under the rectangular function.

Caleb's response is shown in Figure 4.6. His plot is different from the plots made by the other participants; however, he demonstrates the same thinking as the others about the limits of the plot of the integral. He said,

> So, $s_1(t)$ will be the integral of the signal. So, we got the signal, it's a rectangle signal. It's from negative 1 to 1 in time domain. We want and there's an amplitude, is 1. $s_1(t)$ should be the integral, so it should be integrated from negative 1 to 1, and the function is rect(t/2) dt. So ... then, divide by that. So, rectangle function is ... . So, from negative 1 to 0, it is ... . Well ... It looks like the negative t dt plus 0 to 1 ... no. It's just 1 from ... and 1 dt, so it's t ... from negative 1 plus zero to 1. The t, it's from zero to 1. So, basically, this looks like ... negative 1 to 1 of T. This looks like the triangle shaped here. (Caleb)

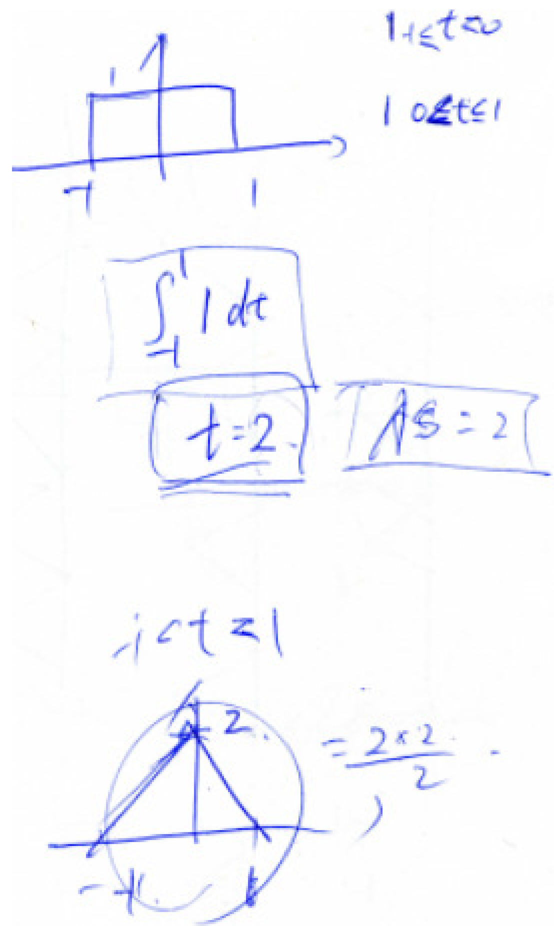

Figure 4.6. Caleb's response for area under the rectangular function.

4.2.1.2 δ(t) is a Function Like x(t) whose Position Varies According to Whatever Value *t* Takes on (SRO2)

δ(t) and δ(ω) are impulse functions centered at $t = 0$ and $\omega = 0$ respectively. The data suggested that the participants thought an impulse function δ(t) on the time axis can be placed according to whatever value t takes on just like in any function x(t). The participants did not demonstrate this problematic reasoning when they used a shifted impulse function. For a shifted impulse function e.g., δ(t-1), the participants were able to identify the position of the impulse on the x-axis, e.g., at t=1. To demonstrate this reasoning, examples of Carl's and Caleb's responses while talking about the Fourier transform of 1 are presented in this section.

> If we're doing one, which becomes two pi Dirac of omega, I guess then that all frequencies would be present, but a Dirac only takes place at one frequency, because I believe a Dirac function has a-- it's defined as having like a width of zero over an infinite height, so it has an area of one, but since this has two pi in front of it it would just be a Dirac with an area of two pi at one specific frequency, and I think that would depend on what time we have. But, I mean, it'll be zero for most frequencies except for omega. (Carl)

> And v3(t) is equal to 1, which is a constant in time domain. And in frequency domain, Fourier transform is 2πω, so, yeah, it has the frequency omega. (Caleb)

4.2.1.3 The Product of a Function and an Impulse Function is a Constant (SRO3)

The product of any function and an impulse function is an impulse function scaled by the constant value of the function at the point where the impulse is located. The data recommended that the participants thought that the product of any function and an impulse function is a constant function. For example, when asked to plot and explain the Fourier transform of $z(t) = t^2\delta(t-1)$, Luke (Figure 4.7) said,

> Okay, so in this case, for z(t), because that's an impulse response we know that it only happens at that point in time. So this has a time delay and so we have to wait until one, and then when we get to one that's when we turn off-- or when we turn

on and it goes immediately off. So that means we're looking at zero, one, and then I'm just gonna kind of ghost out what the picture looks like. We know that we have a magnitude of one. Well, it happens at one because z-- oops-- z(1) equals one squared times one. So we just get one. But we're only doing the Fourier transform of that particular point ... the Fourier transform of one should be one. (Luke)

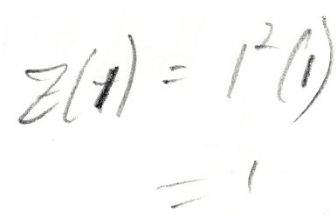

Figure 4.7. Luke's working for simplifying the equation $z(t) = t^2\delta(t-1)$.

4.2.2 Frequency Analysis (FA)

The explanation of the problematic reasonings related to frequency analysis content area and examples from the data that suggest the presence of the problematic reasonings are presented in this section.

4.2.2.1 Periodic Signal in Time Domain is also Periodic in Frequency Domain (FA1)

If a signal is periodic in one domain (time or frequency), it is discrete in the other domain (time or frequency). The data proposed that the participants thought a periodic signal in the time domain is also periodic in the frequency domain. A few examples demonstrating this problematic reasoning are presented in this section.

The participants were given an aperiodic rectangular function and a periodic rectangular function shown in Figure 4.8 and were asked to explain if the knowledge of the frequencies present in a periodic signal help to determine the frequencies present in a corresponding aperiodic signal. Some participants replied that making a signal periodic would make its corresponding Fourier transform periodic too. A few responses are given below.

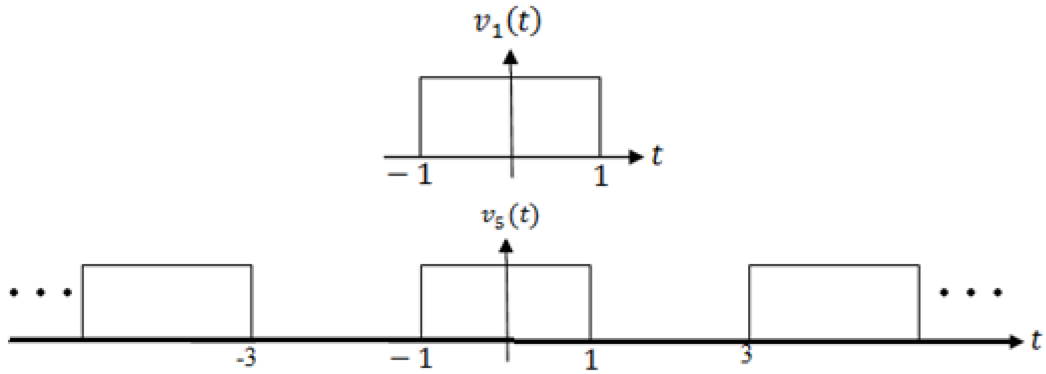

Figure 4.8. An aperiodic rectangular function and its corresponding periodic rectangular function used in a couple of questions in the protocol used for this study.

> Generally I think if something's periodic in the time domain it's probably also periodic in the frequency domain. Like if we have-- this is an impulse train I believe, right? ... . Then the impulse train in the frequency domain is also an impulse train with a different time shift and a different area, so knowing that this one becomes periodic and knowing that to get this you just convolved this with a Dirac function so it gives you another periodic single, then that would make me think that this would also be periodic ... . I'm drawing in dots because I'm not entirely positive where that next one is. I just know what it's supposed to look like. And then if that was at eight pi then you would have another one at negative

eight pi as well, because it's going to repeat periodically. And because it's a sinc function there actually will be a lot of aliasing, and so you just eventually have a train of sync functions, but knowing what frequencies are present in this first sync can help you determine the spacing for the rest of them. (Carl, Figure 4.9)

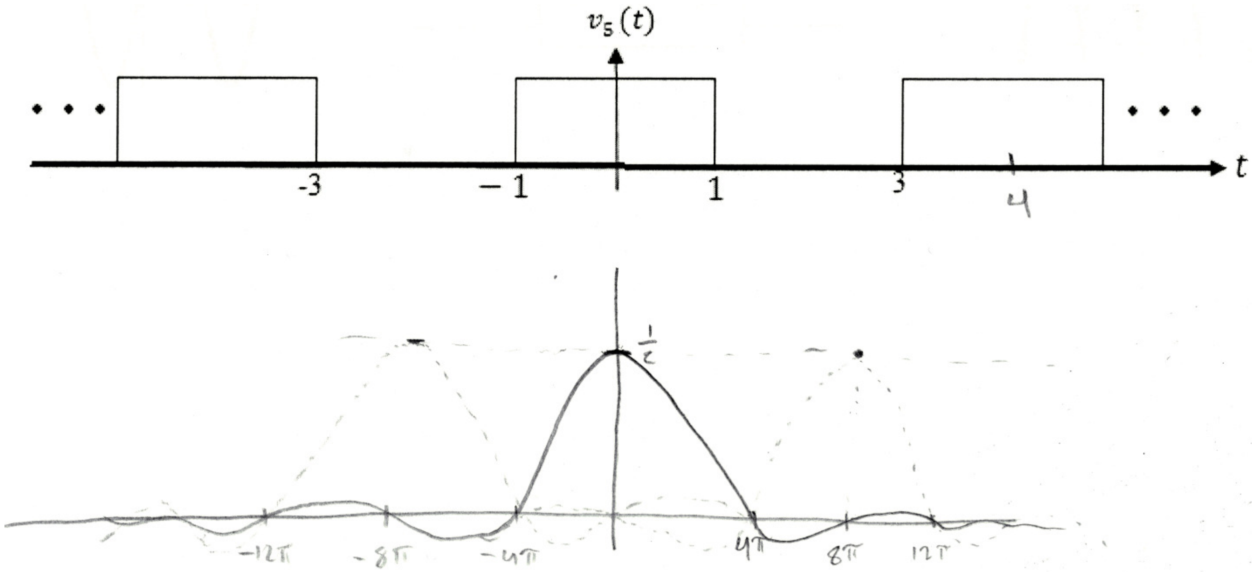

Figure 4.9. Carl's drawing of the spectrum of the periodic rectangular function.

If you know the Fourier transform of this one, you just make copies of the Fourier transform at the given intervals ... . Okay, so superposition theorem, we are applying again the superposition theorem here. (Erin)

If you have aperiodic function and you know the frequencies within that, once you make it periodic you can just multiply those frequencies by whatever the period is to help you find the period, I mean, the frequencies within the other signal, the periodic signal that you created. (John, Figure 4.10)

John used the word "multiply" to mean "create copies" of the signal.

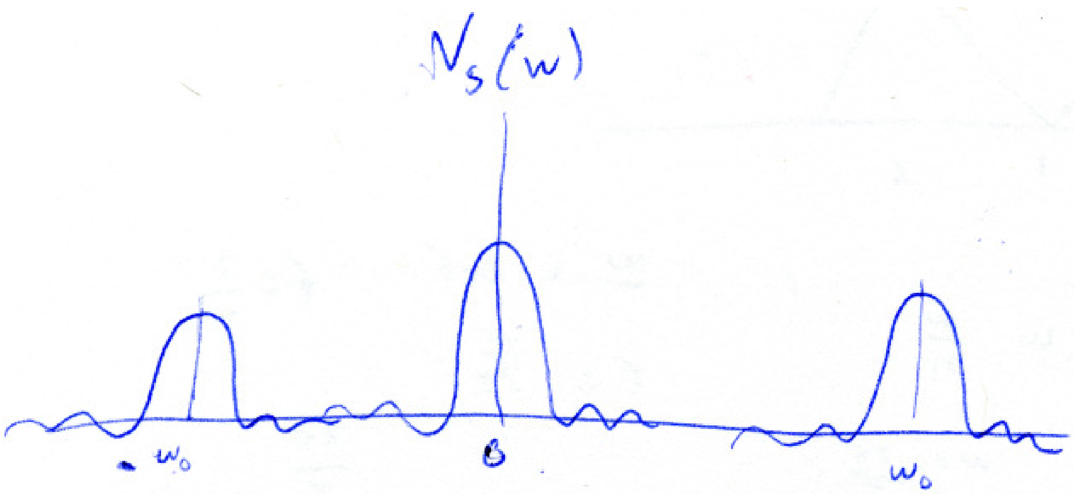

Figure 4.10. John's drawing of the spectrum of the periodic rectangular function.

For the $V_5(F)$ of the $v_5(t)$, it should use the superposition principle. It should have like the sum of the-- the sinc function added up together. And it will-- yeah, it will overlap together at-- and where-- canceled in some parts, so at the end, it should look like ... maybe it looked like that. So, it should represent one of these-- the signal, repeated periodic signals. (Caleb, Figure 4.11)

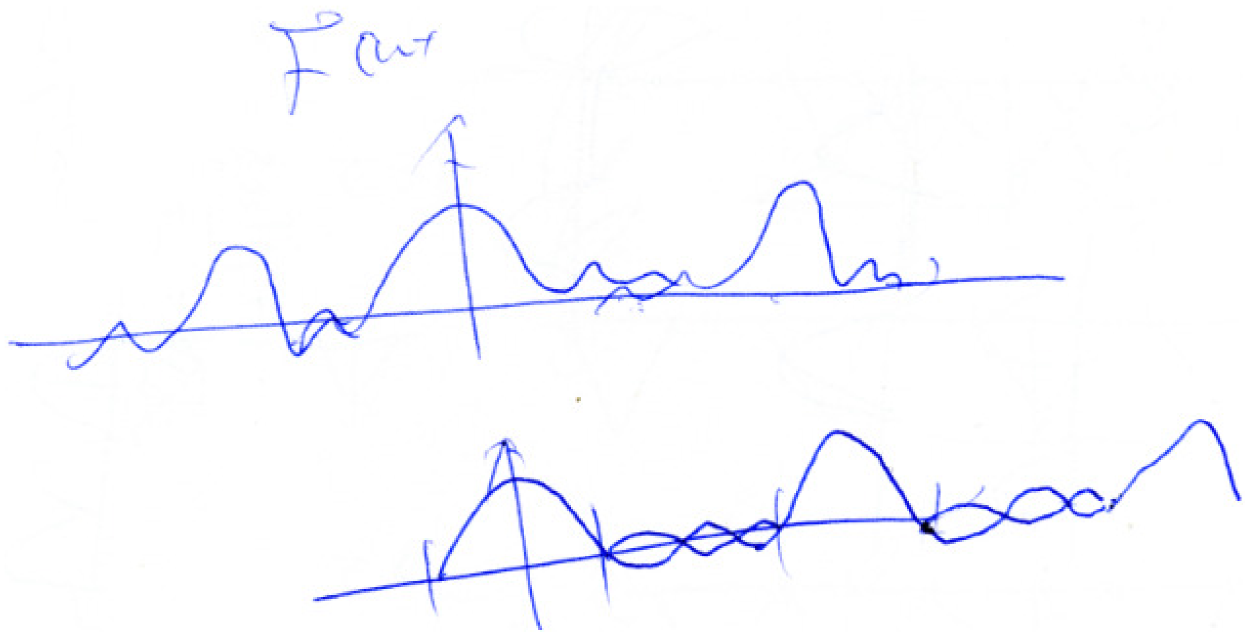

Figure 4.11. Caleb's drawing of the spectrum of the periodic rectangular function.

Oh, absolutely. If you have this and now you make a periodic extension of it you're doing the same thing with the transform. It's going to be a periodic version of that sync function. But, again, going back to the superposition, it's not going to show up like this. You're going to have to find where they overlap, and it would be one larger sync function that would capture the frequencies that all of them captured. ... Oh. Because I feel like Fourier transform always matches with what you have, so if this is a single rectangle this would be a single sync function. If this is periodic this is also going to be periodic. They always coincide. You're not going to have a rectangle that is finite and then an infinite train of sync functions. They have to coincide, if that makes sense. (Emily, Figure 4.12)

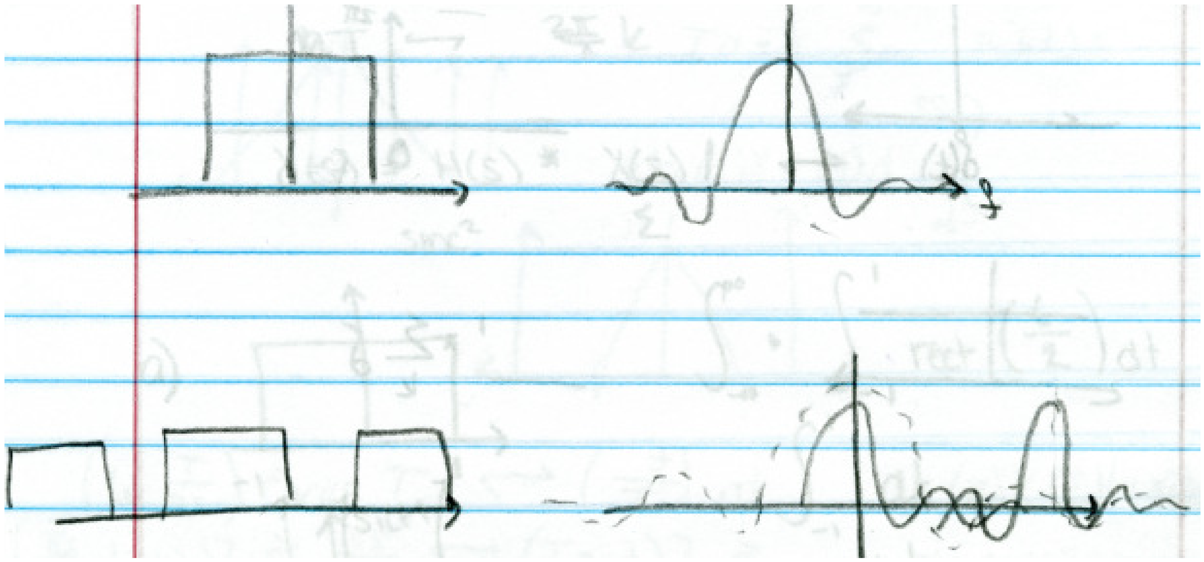

Figure 4.12. Emily's drawing of the spectrum of the periodic rectangular function.

Yeah, so, we have a set of frequencies in $v_1(t)$, so we know the range of frequencies. So, the difference between the highest frequency and the lowest frequency, so that will give us the range of frequencies. And then, once you take the difference between the highest frequency and the lowest frequency, you have the bandwidth, so how long the signal is. And then, depending on the period at which it occurs, so, at T equals zero, it's going to be the exact same as $v_1(t)$. So, it's going to share the same set of frequencies. But when you're looking at the next signal occurring at after one period, your center is going to be different. So, this is centered at zero. This is not centered at zero. This is, I'm guessing, centered at 4. So, it's going to contain, so, you're going to look at the frequency at which the 4 corresponds to, and then it's going to be within plus or minus the range of frequencies that you have in $v_1(t)$ ... (Justin)

When asked about the frequency of $v_4(t) = 0.504 \sum_{k=-\infty}^{\infty} \frac{1}{1+j4k} e^{jkt}$, Emily said

> That's an ugly signal. So what I have here in the time domain is a repeating function, and it goes on infinitely, so when I flipped it, because it's a train I'm assuming that I'm going to get some kind of a impulse train in the frequency domain. So because of that I would know that it would be periodic and it would capture frequencies at each period, and it would repeat on forever ... . Because of the summation I'm assuming that when I took the inverse transform of it or the transform to the frequency domain I would get a pulse train. It would be a summation in there, and because I would have that summation with a K and some kind of delta-- and then that would either be convolved or multiplied with some other function, probably whatever this would transform to, I would know that it would repeat, and it's going to be the inverse of this period, two pi over whatever that period is. And it would capture those frequencies, so whatever my signal was it would probably look like this over and over again. (Emily)

4.2.2.2 Signal Representation in Time Domain is Same Representation in Frequency Domain (FA2)

The data suggested that some participants thought a signal's representation in the time domain is the same as the signal's representation in the frequency domain. A few examples that demonstrate this problematic reasoning are presented in this section.

The participants were asked to explain the values of the frequencies present in an impulse function and in a constant. The participants demonstrated problems in explaining the values of the frequencies present in the given signals despite the availability of the

Fourier transform table and related formula sheet. An impulse function has all frequencies, but because an impulse function is located at $t = 0$, participants replied that the impulse function has zero frequency. Additionally, a constant in time means zero frequency but the participants said that a signal if constant in time is constant in frequency too and so contains all the frequencies equally. Section 4.2.1 demonstrates that the students displayed problematic reasonings in the use of an impulse function. The problematic reasonings used to explain the frequencies of an impulse function could also be related to students' understanding of an impulse function in general.

> If this were ... well, since this isn't present, this one isn't present at any other values besides zero. When this goes to the frequency domain, it'll just be 1, and, well, actually, this is a better example, I think ... with the shifting. If time shifting property shows that if I have a signal G of T minus T naught and I go to the frequency domain, it's going to have some value, the frequency representation of the signal multiplied by essentially a frequency that it's present at and shifted over. And I guess just my understanding of that, of some of these properties that this is essentially the delta function of T minus zero, so in the frequency domain, that will be E to the negative J zero-- or no, no, omega times zero, which is 1. And so, it's only going to be present at one frequency and it's not-- it's not continuous. It's-- I already know that the delta function is a discrete function. And 1 here, that will move on infinitely in the time domain, so when it goes to the frequency domain, it will remain infinite in length for frequencies, and then same with this one. It's-- if it's continuous for all frequencies in the time domain, it's going to be continuous for all frequencies in the frequency domain. (Tom, Figure 4.13)

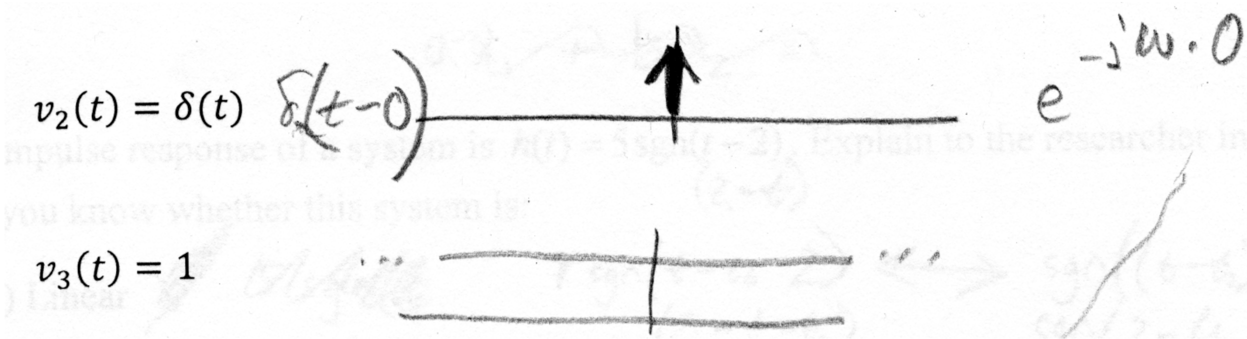

Figure 4.13. Tom's drawing of the spectrum of the given signals.

Frequency is not present because this one is an impulse function ... they only have one value and this one is at when T equals zero. So it doesn’t show frequency. (Matt)

For $v_2(t)$ is equal to an impulse $\delta(t)$ it would just be an impulse at zero, so there would be no frequencies present. It would only be an impulse at zero ... Just because of what a delta function looks like. If that's $v_2(t)$ I know that that's just an impulse there. If I put that in frequency domain that's just going to be one, and so it would be frequency with the amplitude of one, but it would still be a delta function, I believe. And since it's at zero-- and if that's time then the inverse two pi over zero-- oh, it would not be zero, because that would be I guess infinity. Hmm. Threw my explanation out. I still don't think there would be any frequencies present really. (Emily)

Additionally, in a question Luke while trying to find Fourier transform of $z(t) = t^2\delta(t-1)$ said that $z(1) = 1$ and that "Fourier transform of one should be one."

Furthermore, in response to a question to find Fourier transform and Fourier series of $x_4(t) = e^{-j\frac{\pi}{2}}[\delta(t+\pi) - \delta(t-\pi)]$, Jake found that the Fourier transform of $x_4(t)$ is a sinusoidal signal and said that "the Fourier series should have same graph as the Fourier transform-- because this signal is periodic". Lily did the same thing. She looked at this question and said "I was thinking it was already a Fourier transform" and then she drew this function as Fourier transform of this function.

4.2.2.3 A Constant in Frequency Domain Means No Frequency (FA3)

A constant in the frequency means all frequencies present equally. The data suggested the participants utilized the problematic reasonings when explaining what frequencies were represented by a constant in the frequency domain and thought that no ω means no frequency. The participants were asked to explain the frequency components of an impulse function and were given the Fourier transform table too. They identified the Fourier transform of an impulse function from the Fourier transform table but had trouble with explaining what the constant means in the frequency domain, as the constant representation does not have a variable ω in it.

> So, $v_2(t)$ is equal to $\delta(t)$, which is an impulse in time domain, and for the Fourier in the frequency domain, it's just 1. So, I don't think there is a frequency represented in this signal ... Because there's just no omega in the frequency domain, just a 1. (Caleb)

4.2.2.4 Phase Shift Means Shifting Phase Plot of Signal in Frequency Domain (FA4)

Phase shift is time shift in time domain calculated in terms of angle. The data recommended that the participants interchanged the concept of time shift in the time domain with the concept of phase shift in the frequency domain. A few examples that demonstrate this problematic reasoning are presented in this section.

The participants were given the signal $h(t) = \sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$ and were asked to explain the concept of the time shift and the phase shift using $h(t)$ as an example. Following are some of the responses:

> And then for a phase shift ... in the frequency domain, it would look just like the time shift does in the time domain. (Jim)

> A time shift is a phase shift in frequency, which is, again, shifting it over. But that’s in the frequency domain. (Megan)

> So, a phase shift would be when the signal would be shifted on the frequency axis, so we'd get frequency, and then, if the frequency ... or if the frequency of this is ... So, if it's-- the frequency is pi over 2, then the frequency shift would just shift it either higher or lower. (Lily)

> The phase shift would happen almost exactly the same with respect to time shift, what happens in the frequency domain, so it's just a different domain of time. It's the same, the same idea of the system, but because of the new angle being

> introduced, the phase has changed, whether it's in the future or if in the past or delayed, so coming later ... what we would be doing is kind of shifting this signal just along where now our x-axis is in frequency, so it's in hertz or in radians, and we're looking at that going back and forth. (Luke)

Megan while trying to find Fourier transform of $x_4(t) = e^{-j\frac{\pi}{2}}[\delta(t+\pi) - \delta(t-\pi)]$ said,

> I'm pretty sure it's just a phase shift in the frequency domain ... because now I've got the time delay. So looking at them one at a time, I should have one E to the J pi minus J pi, possibly. So from there, I can then draw that at least. I'm still trying to draw. I've got 1 to the E to J pi, so that's going to be a 1, but it's phase shifted over to pi. So that's a delay in the time domain. It's a frequency shift. It's a shift in the frequency domain. So if I'm shifting it over pi, it should just start at pi. I'm not sure what else it could possibly do there.

4.2.3 System Analysis (SA)

The explanation of the problematic reasonings related to System Analysis content area and examples from the data collected that suggest the presence of each problematic reasoning are presented in this section.

4.2.3.1 Convolution and Multiplication are Interchangeable (SA1)

The data suggested that the participants interchanged the concept of convolution with multiplication and illustrated two different reasons behind this incorrect interchange.

Firstly, the process of convolution involves multiplication of the two signals for different regions of time, and secondly, convolution in the time domain is multiplication in the frequency domain and vice versa. A few examples that demonstrate this problematic reasoning are presented in this section.

The participants were given two same signals (Figure 4.14) expressed in the time domain and then in the frequency domain and were asked to explain if the convolution of the two signals would result in the same signal in both domains or not. Following are some responses:

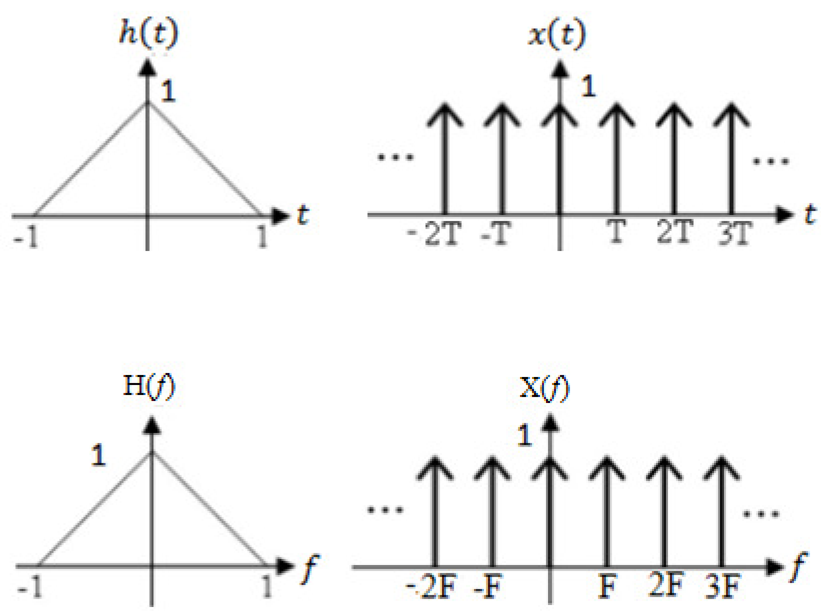

Figure 4.14. Two same signals given to the participants to discuss the convolution result of the signals in the time and frequency domains.

Because it's in the frequency domain, so the convolution is basically, is H(f) times X(f). So, simply multiply these two, and you only got the one shaped at the origin, so there's no others at the rest. So, it's just one of these triangles. (Caleb)

Yes, the plot would be very different, because they're already in frequency domain ... it's multiplication in time. (Emily)

So instead of looking at this in a time domain, we're looking at it in the frequency domain ... the way I remembered it was convolution in the time domain is multiplication in the frequency domain ... The first thing I thought of was, you know, they're going to be the same. But that's because I didn't really read the-- I just saw the shapes ... So we know that they're inversely proportional for frequency and time. Well, we know now that we can just multiply this. Your H signal with your X signal. So it's not going to look like what we did for part a ... So one's multiplication, one's convolution, so that would produce a different output. (Justin)

Yeah, I think it will change ... Because the Fourier transform is changed from time domain to the frequency domain. I think h(t) will give the range of the y(t) ... So it's from -1 to 1 ... y(t) would be from -1 to 1 too ... Because there's a filter so it will filter out the parts that doesn't include in that. (Matt)

4.2.3.2 Concept of Time-invariance of System is Interchangeable with Literal Meaning of Time Invariance (SA2)

If the output of a system is $y(t)$ for an input $x(t)$, then the system is time-invariant if the output is $y(t-t_o)$ for the input $x(t-t_o)$. The data illustrated that the participants thought that the concept of the time-invariance of a system could be explained with the literal meaning of time-invariance. This reasoning was suggested from the participants' responses to the question about determining if the given system (Figure 4.15) was time-invariant. A few responses of the participants are presented in this section.

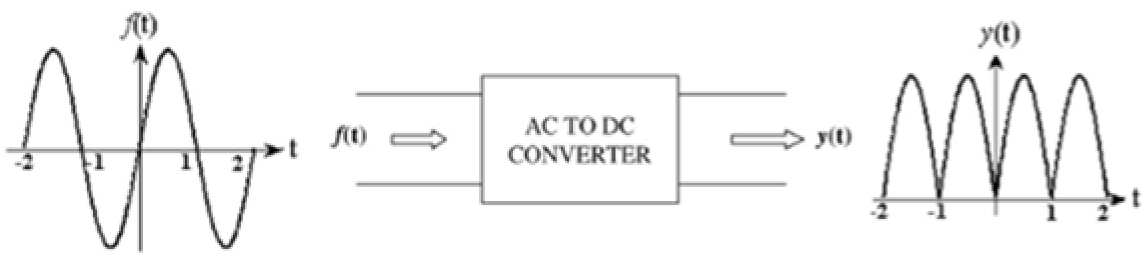

Figure 4.15. Picture of the ac-to-dc converter given to the participants to determine if it is a time-invariant system.

> Time-invariant, it would be the same shape output but not necessarily the same time. And not time-invariant would be the output would be the same at the same time. (John)

> For time-invariance it follows the same what you get in you get out at the same time. The only thing is it's rectified, so its negative components are now positive,

> but it follows the same period. It's not shifted at all. It's not changed. It hasn't moved back. So here it's easier to see that it would be time-invariant. (Emily)

> It doesn’t get affected by the time inside the converter so it can be time invariant. (Matt)

> I think if I remember correctly the rule for time-invariance if you're looking at an equation is since T is not being multiplied by anything it's time-invariant ... Because the system that comes out is still periodic then that would lead me to believe it's time-invariant. (Carl)

4.3 Difference between Problematic Reasonings among Students with Different Academic Statuses

This section presents the answer to the second research question of the study, "How the problematic reasonings differ after the students take more courses that require prior knowledge of Continuous Time Signals and Systems course content?" Nineteen participants (Table 3.2) were interviewed for this study. Of the nineteen participants, eight participants belonged to CTSS-only group (had taken only one Continuous Time Signals and Systems course) and eleven participants belonged to CTSS-plus group (had taken related courses subsequent to Continuous Time Signals and Systems).

The comparisons given in Figures 4.16 and 4.17 are based on the number of participants in each group using a certain problematic reasoning (section 4.3.1) and the number of problematic reasonings demonstrated by each group (section 4.3.2) respectively. The two different comparisons show how that the number of problematic reasonings per group and the number of students per group that demonstrate a particular problematic reasoning are distributed. This is important because if one group represents ten problematic reasonings and only one person in that group is using that particular reasoning, then the use of reasoning may not be a representative of the whole group.

Rather than comparing the number of the problematic reasonings employed with the number of participants in each group employing that particular reasoning, I am comparing the number of the problematic reasonings employed with the normalized (standardized) proportion of participants in each group demonstrating a particular reasoning. There are two reasons for this choice. First, if all the participants were given an equal chance to demonstrate a particular problematic reasoning, and all demonstrated that particular problematic reasoning, the number of problematic reasonings demonstrated by the CTSS-plus group would have been more as the number of students in this group is higher. Second, for questions related to topics similar to the ones in Continuous Time Signals and Systems courses, there is always more than one way to solve a particular problem. If only participants from one group employed a certain problematic reasoning, there is no certainty that the other participants would not have employed the same problematic reasoning, if they had chosen to solve the same question in the same way. While comparing the differences in the reasonings employed by each group, I acknowledge that not all the participants got an equal chance to reveal a particular reasoning, as everyone preferred their own choice of methods to respond to the problems. In both cases, I normalize the number of students so that the results indicated would match the results if the number of students in each group were the same. Figure

4.16 is the number of students displaying a particular reasoning in each group (section 4.3.1) and Figure 4.17 is the difference in number of problematic reasonings displayed by each group (section 4.3.2).

4.3.1 Number of Problematic Reasonings Demonstrated by Normalized Proportion of Individual Participants in Each Group

Figure 4.16 shows the normalized proportion of students from each group that demonstrated a particular problematic reasoning (section 4.2). Figure shows that the problematic reasonings demonstrated more in the participants from the CTSS-only group are:

1) Convolution and multiplication are interchangeable (SA1)
2) Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance (SA2)

Figure 4.16 shows that the problematic reasonings used more often by the participants in the CTSS-plus group are:

1) $\delta(t)$ or $\delta(\omega)$ are functions like x(t) which varies according to whatever value *t* takes on (SRO2)
2) The product of any function and an impulse function is a constant (SRO3)
3) A constant in the frequency domain means no frequency as it has no ω in it (FA3)
4) Phase shift means shifting the phase plot of a signal in the frequency domain (FA4)

Figure 4.16 shows the problematic reasonings demonstrated almost equally (at least 40% or more often) by both groups are:

1) Any property of a signal is limited within the duration of the signal itself (SRO1)

2) A periodic signal in the time domain is also periodic in the frequency domain (FA1)
3) Signal representation in the time domain is the same representation in the frequency domain (FA2)

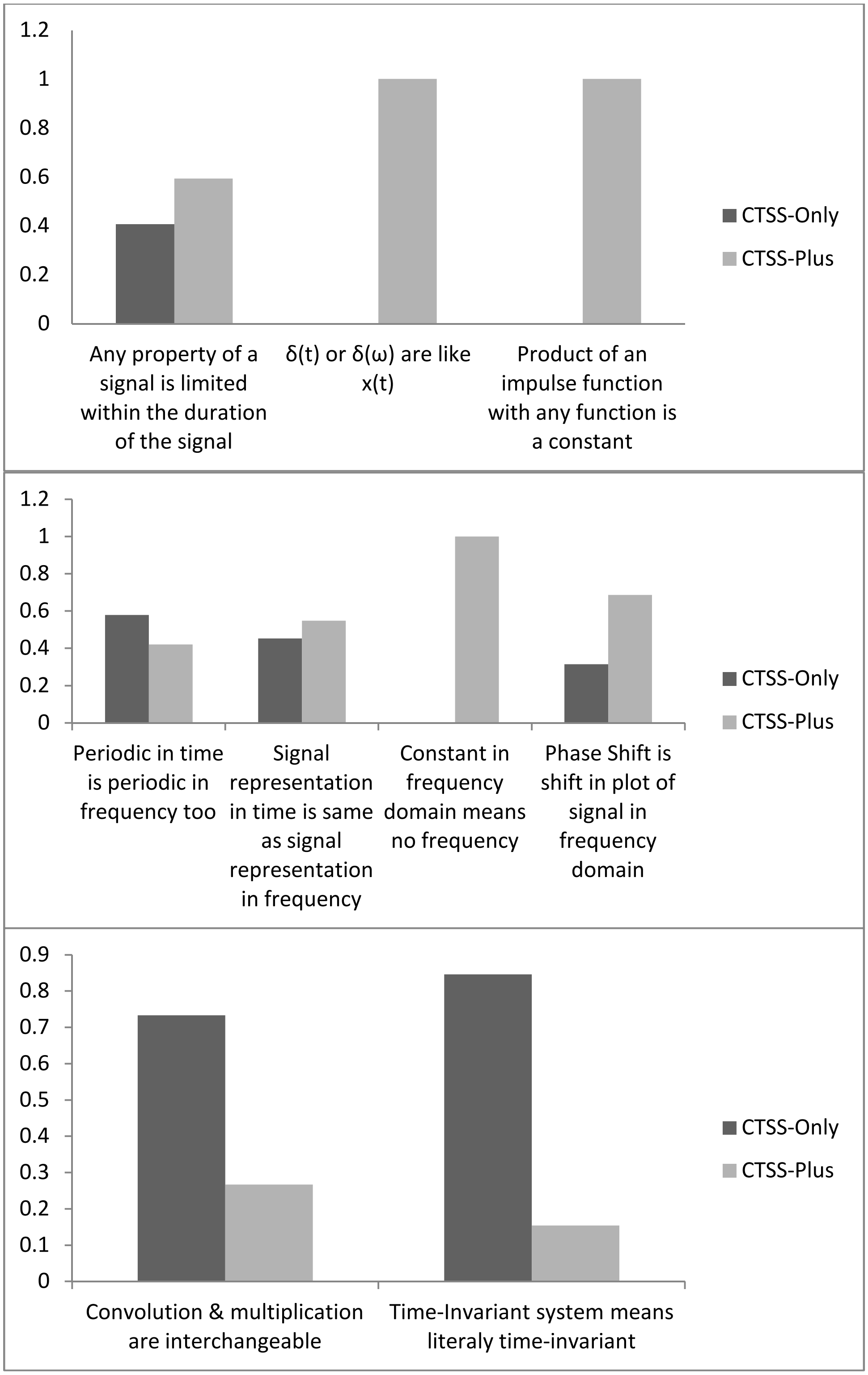

Figure 4.16. Normalized proportion of the number of students in each group employing a particular problematic reasoning.

4.3.2 Normalized Proportion of Problematic Reasonings Demonstrated by a Group Collectively

This section presents the normalized proportion of the number of problematic reasonings (presented in section 4.2) demonstrated by the CTSS-plus (Table 3.2) and CTSS-only (Table 3.2) groups. Figure 4.17 demonstrates the proportion graphically. The discussion of these proportions will be presented in section 5.2. Figure 4.17 shows that the normalized proportion of the number of problematic reasonings demonstrated more often by the participants from the CTSS-only group are:

1) Convolution and multiplication are interchangeable (SA1)
2) Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance (SA2)

Figure 4.17 shows that the normalized proportion of number of problematic reasonings used more often by the participants in the CTSS-plus group are:

1) $\delta(t)$ or $\delta(\omega)$ are functions like $x(t)$ which varies according to whatever value *t* takes on (SRO2)
2) The product of any function and an impulse function is a constant (SRO3)
3) A constant in the frequency domain means no frequency as it has no ω in it. (FA3)
4) Phase shift means shifting the phase plot of a signal in the frequency domain (FA4)

Figure 4.17 shows that the normalized proportion of the number of problematic reasonings demonstrated almost equally (at least 40% or more often) by both groups are:

1) Any property of a signal is limited within the duration of the signal itself (SRO1)
2) A periodic signal in the time domain is also periodic in the frequency domain (FA1)
3) Signal representation in the time domain is same representation in the frequency domain (FA2)

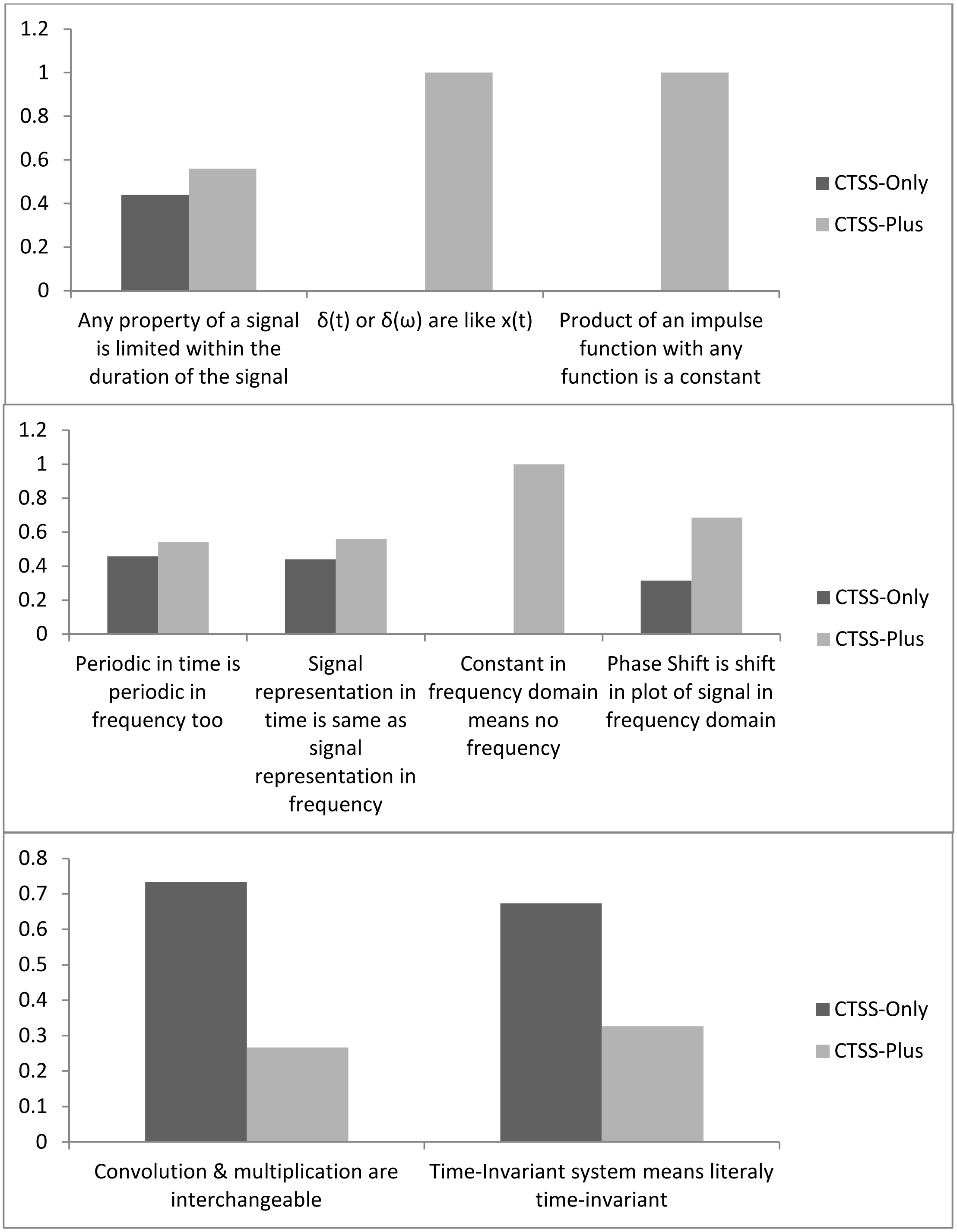

Figure 4.17. Normalized proportion of a particular problematic reasoning employed by each group collectively.

The data represented in Figures 4.16 and 4.17 are almost similar. This shows that the problematic reasonings that are more prevalent in one group (Figure 4.17) are prevalent among the participants of that group as well (Figure 4.16).

4.4 Additional Findings

The aim of this study was to identify students' reasonings that can cause difficulties in conceptually learning topics taught in Continuous Time Signals and Systems courses. These reasonings are presented in section 4.2. These responses were labeled problematic reasonings because i) they led the participants to give incorrect, inappropriate, or incomplete responses during the interviews, and ii) the participants were very clear in describing their thoughts behind these responses. However, there were several instances in the collected data where the participants' responses were incorrect, inappropriate, or incomplete but they did not explain their reasonings. These responses are labeled as mistakes (section 4.4.1) or missing conceptual knowledge (section 4.4.2) based on the distinction given later in this section. I am presenting the prevalent mistakes and missing conceptual knowledge from the data as 'additional findings' because in my opinion prevalence makes them attention worthy for both the instructors and learners of these courses.

Additionally, the data suggested that in general the participants preferred to solve a question using mathematical equations as compared to solve by making graphs. I argue that understanding this general trend is important for design of effective instruction for this course. The details and examples from data demonstrating this trend are presented in section 4.4.3.

4.4.1 Mistakes

The incorrect responses in the data for which enough evidence for the participants' reasonings behind them was not available are called mistakes. As suggested by the collected data, the situations where most of the participants made mistakes are given below. The description of these situations and the examples from the data supporting the presence of mistakes in these situations follow the list.

1. Engaging with the powers of exponential functions
2. Translating a mathematical equation
3. Engaging with a unit step function
4. Engaging with an impulse function
5. Performing time shift and time scale operations combined
6. Interchange similar terms and concepts

4.4.1.1 Engaging with Powers of Exponential Functions

The data illustrated that the participants made mistakes in engaging with the powers of exponentials. A few examples demonstrating this mistake are given in this section.

After expanding $v(t) = \cos\left(t + \frac{\pi}{4}\right) + 3\sin(7t)$ using inverse Euler's identity, Bill could not handle j in the denominator as well as in the multiplication of the two exponentials as shown in Figure 4.18 and said,

> Okay, so this guy, if I'm gonna do Fourier series with it, I wanna put these both in the Euler's, the e(s), 'cause I wanna try to combine 'em so that I can actually get 'em in a workable form for this. So cosign, that becomes-- so it's e-- actually, it's

on the back here. Just to make sure I'm not doing it wrong. Well, I thought it was on the back. Nope it's not. Okay, from what I remember-- I'm trying to remember what I do with the pi over 4. All right-- man, I don't know what I'm drawing <inaudible> here. Sometimes I just need to-- all right. So cosign, I know, it's e j positive plus e to the negative j, all over 2. So then the t, since it's just one, that's why you just have j. The cosign of pi/ 4, I'm pretty sure it just goes back to, like-- even looking at this table I think it's just the shift. So it'd just be like e to the j pi over 4 on the outside. That's the part I'm struggling with right there. I don't quite remember what you do with the pi over 4. But then for the sign, it's 3 and then it's times all of the e to the ... Caught that. E to the j 7-- well, I guess-- what I need. I'm thinking now do any of the t still? I'm kinda just like dropping it. But-- or would it change? I'm gonna assume later I'm gonna need the t but I'll finish this. And then so sign is e to the minus j 7 all over 2 j. From there I'm gonna try to combine 'em. Solve the point of putting in this form so that you can kinda reduce 'em all down together. Yeah, so you have-- trying to think of the t. Right now, yeah, right now I'm just-- I'm thinking, when I put it in, 'cause it's on one-- I'm trying to get it so I can use this form for the Fourier series. And the a of k is my omega 2 and then I wanna find the phase. So I wanna get it into a form that I can multiply these through and then actually take the integral. J of k, omega naught t. I--yeah, I'm ma leave it like this. This is fine. And then I'm gonna-- so I'm gonna try to simplify a little bit how-- well, I mean, I guess this one, when I pull this out, you coulda left that in-- 2 <inaudible>. J of 1 plus pi over 4. 'cause they just add. And this coulda been e to the j negative 1 plus pi over 4. This is over 2; this is over 2, minus 3/2, e to the negative j 7. Man do I not remember this. Yeah, wasn't good at these in class either. (Bill)

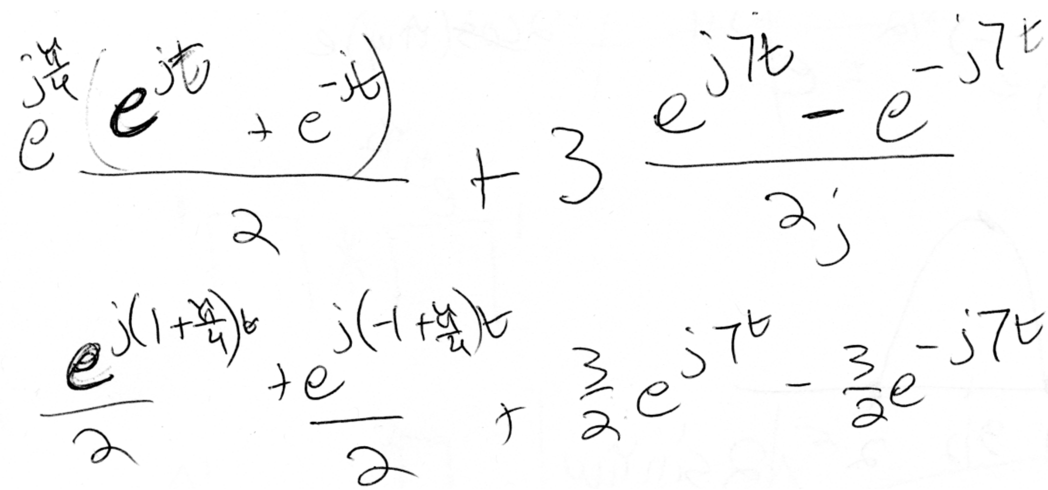

Figure 4.18. Bill's working with powers of exponential functions.

The power of a power is multiplied (e.g., $(e^{j})^{\omega t} = e^{j\omega t}$) and powers are added up after multiplication of two functions with the same bases (e.g., $e^{j\omega} \times e^{t} = e^{j\omega + t}$) which means $e^{j\omega} \times e^{t} \neq e^{j\omega t}$. Bill demonstrated this mistake in engaging with an exponential function while trying to find the Fourier transform of $d(t) = t^2$ shown in Figure 4.19.

Take the integral from negative infinity of t squared. Either do a omega of t. Which I don't think I can do by hand, because you basically are left with the integral of T squared. e to the j omega t, and so, like, you could pull that part out. So you have e to the negative j omega, integral, e to the t. And I know that's a rule. <laughs> But I don't remember it. So yeah. I would just do that, and it would give me my F of T. So that's how I would do it ... I don't remember that integral. That's another one of those computer things. I took calculus, I learned it, and then they're like, "Ah, just use your computer…" for the rest of my career. So it's one of those I haven't done in forever. (Bill)

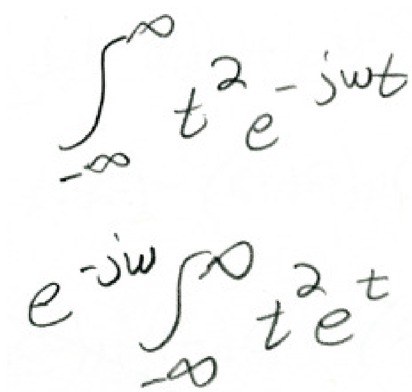

Figure 4.19. Bill's working to find Fourier transform of t-square.

Furthermore, Luke displayed a mistake while trying to find Fourier transform of $x_4(t) = e^{-j\frac{\pi}{2}}[\delta(t+\pi) - \delta(t-\pi)]$ by replacing ω in $e^{-j\omega t}$ with $\pi/2$ instead of using $e^{-j\frac{\pi}{2}}$ as a constant. He said,

> If I'm remembering this right, the first problem would use a Fourier transform ... of this ... and, I mean, you'd be integrating it with-- oops-- with respect to T and omega. This omega would be pi over 2, because that's what it is in x_4. And then, you would just-- you'd have to understand that as T goes from infinity to infinity, especially since we're using delta functions, that they kick on at zero and they're just quick little impulses. (Luke, Figure 4.20)

$$\overline{X}(\omega) = \int_{-\infty}^{\infty} x_4(t)\, e^{-j\overset{\pi/2}{\overset{\downarrow}{\omega}} t}\, dt$$

Figure 4.20. Luke's working to find Fourier transform of $x_4(t) = e^{-j\frac{\pi}{2}}[\delta(t+\pi) - \delta(t-\pi)]$.

Jim, while trying to find Fourier transform of $x_3(t) = \sin\left(\frac{\pi}{6}t - \frac{\pi}{6}\right) + e^{j\frac{\pi}{3}}$, said

Okay, so for this one you can break this up into the two parts, the sine and the complex exponential, and the first way I would do it is I'd take each individual one. So we have the complex exponential, which goes to two pi delta over omega, so minus pi over six, and then there is a time shift, which would give us E to the negative-J ... Okay. We have impulse at pi over six and negative-pi over six ... And then we have one at pi over three by two pi. (Jim, Figure 4.21)

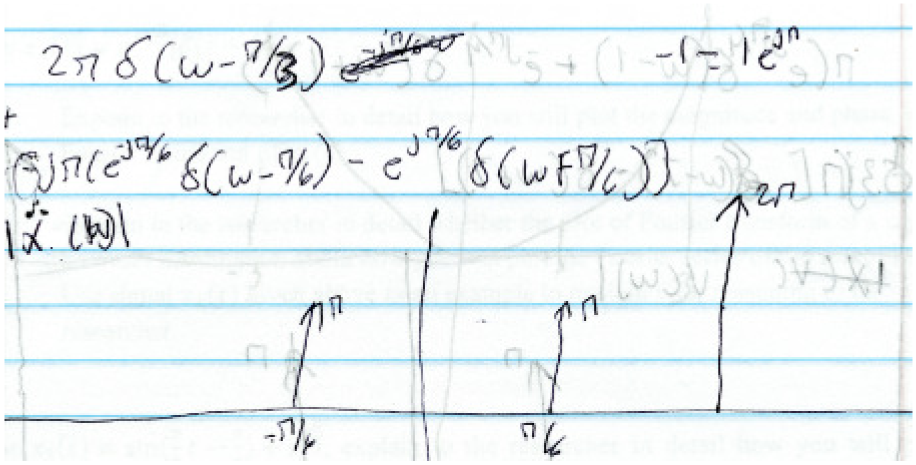

Figure 4.21. Jim's working to find the Fourier transform of $x_3(t) = sin\,(\frac{\pi}{6}t - \frac{\pi}{6}) + e^{j\frac{\pi}{3}}$.

4.4.1.2 Translating Mathematical Equation

The data illustrated that the participants made mistakes in translating a mathematical equation. A few examples demonstrating this mistake are given in this section.

Lily made the graph of t^2 shown in Figure 4.22 and did not realize that t^2 is not zero for negative time.

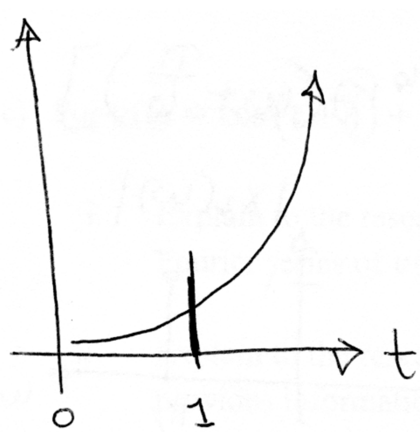

Figure 4.22. Lily's graph for t-square.

Furthermore, the participants were asked to plot the integral of a rectangular function (Figure 4.8) and explain if the plot of the area of the rectangular function helps in determining the plot of area under a periodic rectangular function (Figure 4.8). The correct response was that the area of each rectangle in the periodic signal would be added up with the areas of the rectangles before that and the plot of area of a periodic rectangular function will not be periodic. The plots of the area under the periodic rectangular function drawn by Carl and Matt are shown in Figure 4.23.

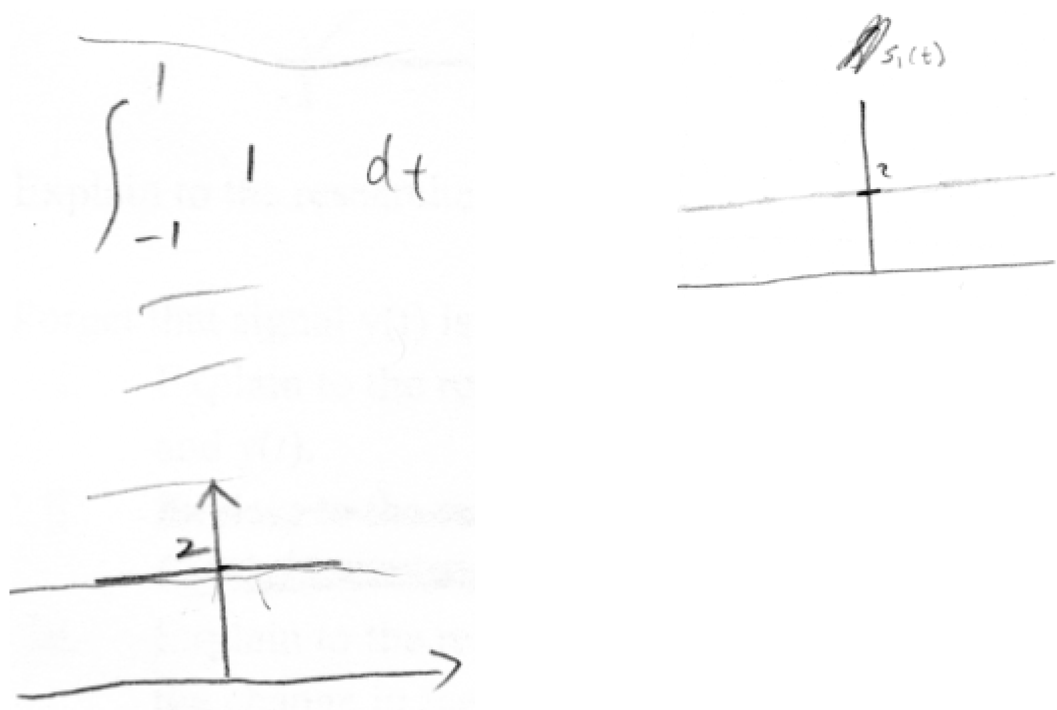

Figure 4.23. Area under a periodic rectangular function drawn by Matt (left) and Carl (right).

Erin correctly identified how he can use the plot of area under the aperiodic rectangular function to plot the area under the periodic rectangular function, but he failed to recognize that area will not be zero before the rectangle centered at 0 as shown in Figure 4.24.

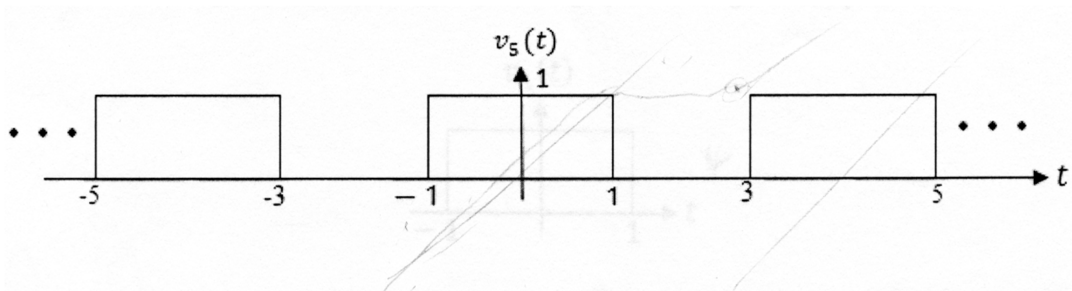

Figure 4.24. Erin's work trying to explain how the plot of area under the periodic rectangular function will look like.

Furthermore, the participants were asked to plot $g(-2t+2)$, where $r(t) = tu(t)$ and $g(t) = [r(t) - r(t-1)]u(-t+2)$, and Luke drew the plot shown in Figure 4.25. He said, "we kind of will do like a parabolic shape in a way."

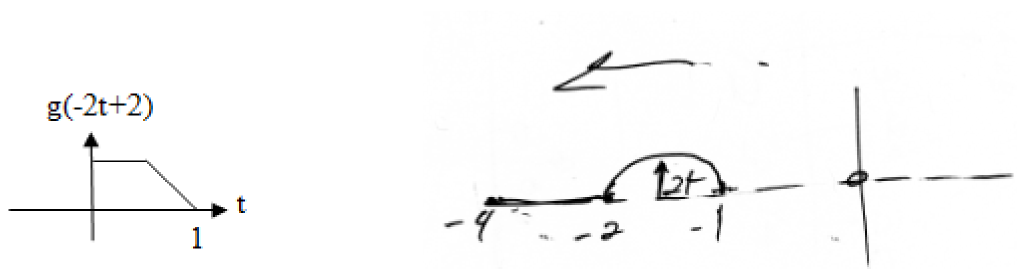

Figure 4.25. Correct plot of $g(-2t+2)$ (left), plot drawn by Luke (right).

In addition, the participants were asked to discuss the frequencies present in $v_4(t) = 0.504 \sum_{k=-\infty}^{\infty} \frac{1}{1+j4k} e^{jkt}$, and Matt said, " Because I think the frequency will be decided by K but since K is a changing value so I don't know how to measure if these frequency keep changing if that still can be counted as a frequency ."

4.4.1.3 Engaging with Unit Step Function

The data suggested that the participants made mistakes when engaging with a unit step function. A few examples demonstrating these mistakes are given in this section.

I asked the participants to convolve an impulse train with a triangular function as shown in Figure 4.14. To convolve the two functions, Tom decided to first find the Fourier transforms of both impulse train and triangular function and then multiply them in the frequency domain instead of convolving them in the time domain. To achieve this, he expressed an impulse train as a unit step function and said, " So, x(t), it's just a pulse. δ(t), I guess the delta function, which are these. These are just-- actually I guess it's a step function. So, if that's u(t), then I go to the frequency domain ...". His working is shown in Figure 4.26.

$$x(t) = u(t) \longrightarrow X(w) = \pi\,\delta(w) + \frac{1}{jw}$$

$$\cdots + \delta(t) + \delta(t+1) \cdots$$

$$h(t) = \Lambda \longrightarrow H(w) = 1 \cdot \operatorname{sinc}^2\left(\frac{w}{2\pi}\right)$$

Figure 4.26. Tom expressed an impulse train as a unit step function.

Lily when trying to find Fourier transform of $x(t) = t^2 u(t) u(1-t)$, replaced limits of integral from 0 to 1 but did not remove *u(t)*s from the expression shown in

Figure 4.27. Because of unit step functions still in the integral she later could not solve the integral.

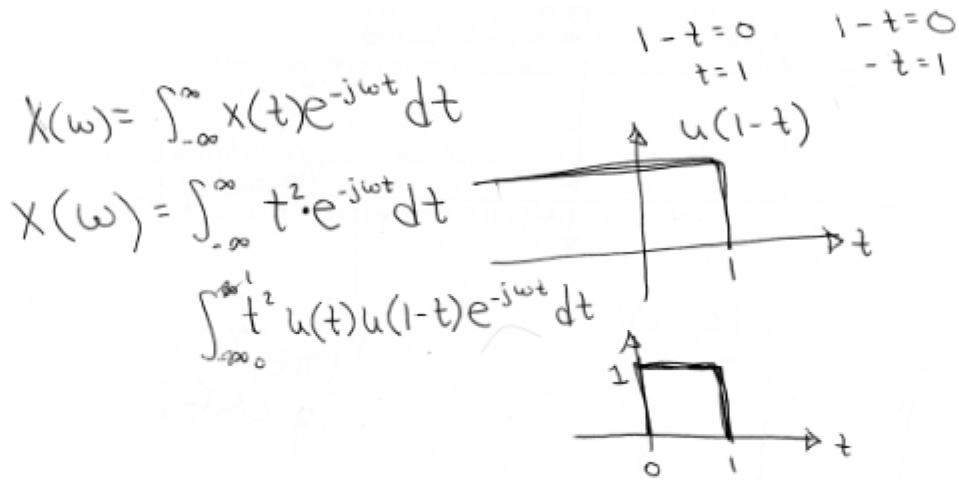

Figure 4.27. Lily's working of using unit step functions inside an integral.

4.4.1.4 Engaging with Impulse Function

The collected data demonstrated that the participants made mistakes when engaging with an impulse function. A few examples demonstrating these mistakes are given in this section. The participants were asked to explain the Fourier transform of $d(t) = t^2$ and then Fourier transform of $z(t) = t^2\delta(t-1)$. A few responses are:

> The delta just moves the t^2 functions. So whatever you had for t^2 it would be moved to-- that's what it is in frequency ... That and the time domain the delta goes to and the frequency domain a constant. And it's got the shift, though. It's got a shift of one and it's a constant times whatever the t^2 is. So it should be the transform of t^2, shifted by one and then multiplied by a constant. (Megan)

> Well, I would just find the Fourier transform and then plug in the value 1 into it and that's my answer, because that pulse means it's 1 at that point only, at T equals 1, and then everything else is worthless. (Bill)

> That that would be the same math here, then plot out. Except the integral would go from one to infinity. (Jim)

4.4.1.5 Performing Combined Operations of Time Shift and Time Scale

The data showed that the participants made mistakes when performing time shift and time scale operations combined. A few examples demonstrating these mistakes are given in this section.

Megan drew the plot of $h(t) = \sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$ as shown in Figure 4.28 and said,

> I'm gonna have it coming up like this, no change to the magnitude. So the next 0 crossing is going to be at pi over 2. So that's 1. So pi over 2 times 2. Yeah, I believe. So I've got it shifted at pi over 2. So I would just solve that for where sign of pi over 2 T minus pi over 4 equals 0. So I should be able to do this in my head. So I'm gonna-- when T equals 0, then T equals 1, T equals 2. So when T equals 0 at T yeah, but it shifts it over 1. So then where T equals 1 and shift it over. So it'd be T plus pi over 4. No, because when T equals 1, it's going to sine is going to equal 1. So I need T equals 2 my-- plus pi over 4 and shifting out like that. Okay. I think that's what I'd do. So 2-- I'm mixing values here, though, I've got a discrete T when it should be-- okay. I'll call that good, though.

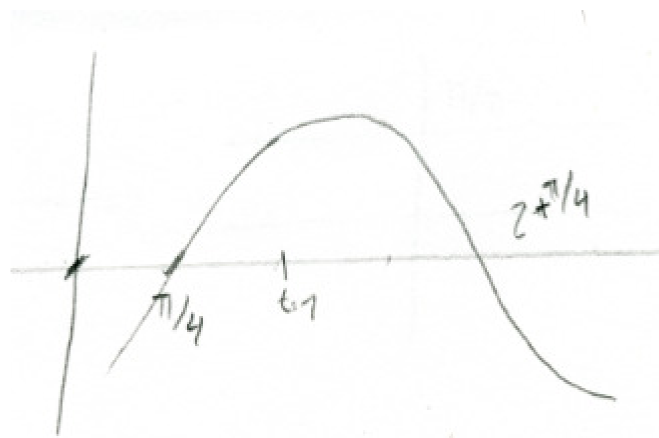

Figure 4.28. Plot of $sin\ (\frac{\pi}{2}t - \frac{\pi}{4})$ drawn by Megan. Instead of π/4, the correct value is 1/2.

The negative sign in sin $(\frac{\pi}{2}t - \frac{\pi}{4})$ means shift the sinusoidal signal towards right. For the plot of sin $(\frac{\pi}{2}t - \frac{\pi}{4})$, Lily said the signal will be shifted at -π/4 as shown in Figure 4.29.

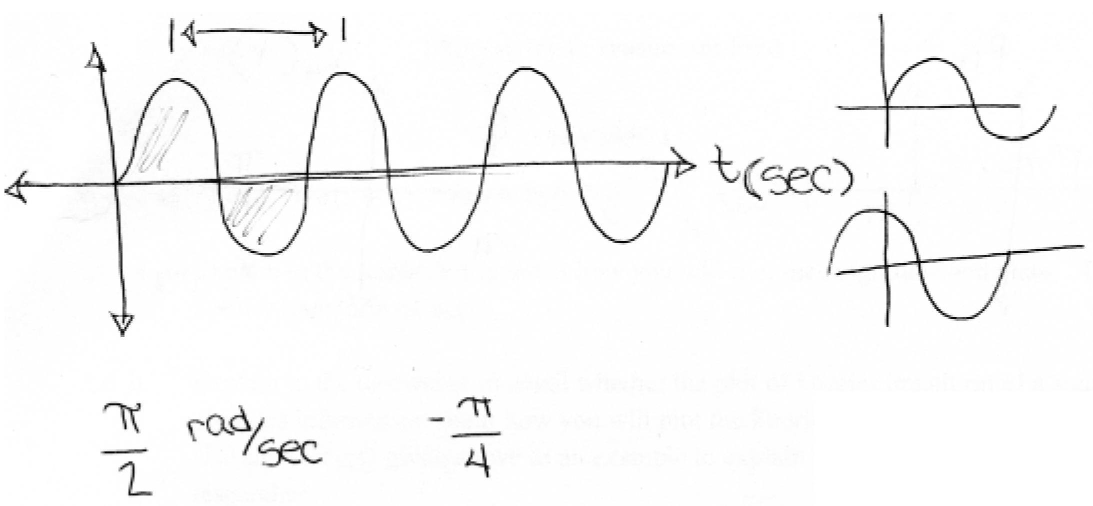

Figure 4.29. Plot of $sin\ (\frac{\pi}{2}t - \frac{\pi}{4})$ drawn by Lily.

Rick shifted the sinusoid signal towards left as well as shown in Figure 4.30.

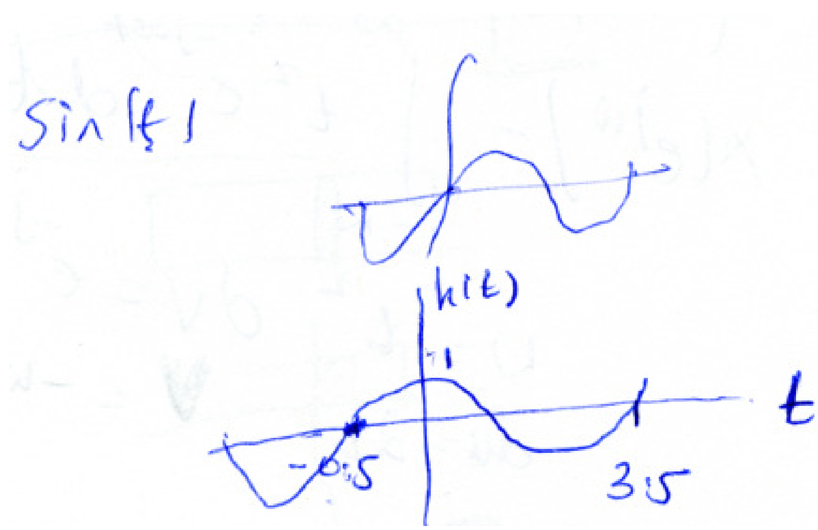

Figure 4.30. Plot of $sin\,(\frac{\pi}{2}t - \frac{\pi}{4})$ drawn by Rick.

In addition, Matt while trying to determine if the ac-to-dc converter shown in Figure 4.15 is linear or not, said the output was a flipped version of the input.

Furthermore, while trying to draw $g(-2t+2)$ where $r(t) = tu(t)$ and $g(t) = [r(t) - r(t-1)]u(-t+2)$, Megan said,

> So negative 2t is going to flip it around, change the magnitude-- yes, flip it around. That's where the negative gets in. The times 2 is going to increase the magnitude and then the minus 2 is going to shift it to the left. Easier if I just think pre-calc.

In the same question, Jim first scaled the function by two and then instead of shifting it by one, he shifted it by two as shown in Figure 4.31.

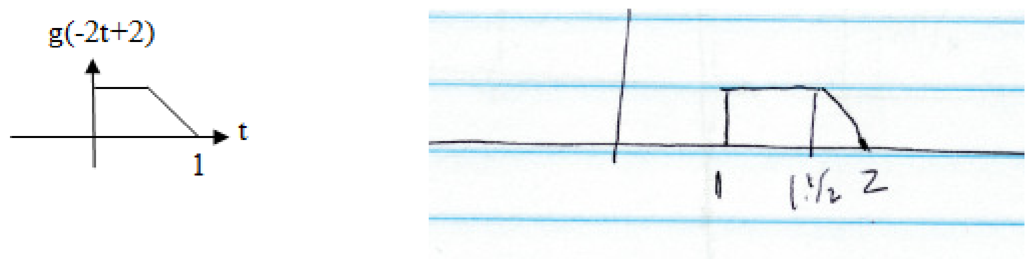

Figure 4.31. Correct plot of $g(-2t+2)$ (left), Jim's plot of $g(-2t+2)$ (right).

Luke and Kevin both got the functions wrong and irrespective of the function they both drew g(-2t) first and then shifted the scaled functions towards left to perform g(-2t+2).

4.4.1.6 Interchange Similar Terms and Concepts

The data suggested that the participants made mistakes of interchanging similar terms and concepts. A few examples demonstrating these mistakes are given in this section.

All signals are composed of frequencies, frequency is the time taken by the signal to complete a period (one cycle), and periodic signal is a signal that repeats itself in regular periods (intervals of time). To answer the question of whether the knowledge of frequencies in an aperiodic signal helps in determining the frequencies of a corresponding periodic signal, Matt related the concept of frequency with the period of a periodic signal instead of using the concept that all signals are composed of frequencies. He said, "I don't think its helpful because one is periodic, the other is not periodic; they cannot be compared ... the way how you repeat it is actually the frequency of this one ... So if it's not periodic it doesn't have a frequency. But the periodic will have one so they cannot be compared and it doesn't help to get a frequency off the periodic function"

Moreover, to plot the area (and nothing related to Fourier series or transform) under a periodic signal with the help of the area under its corresponding aperiodic signal shown in Figure 4.8, the participants used the formula of Fourier series coefficient Ao (Fourier series being completely unrelated in this question) instead of simple integration . Matt said,

"Oh since this one's a periodic, the graph of $s_1(t)$ is a straight line. So this one's supposed to be a straight line too because it's kind of like say a series graph so we only focus on one period of it. So I think we don't actually need to care about the others. We just need to probably make a comment and say this one is a series."

Emily said,

"If this is periodic it would be the same thing as $s_1(t)$ but just repeated, so again if I did the integration I would only pick one area. So whether I did it from negative five to negative three or negative one to one, I would still get the same answer of the amplitude of two, so then I would just go from one to negative one up twice, and then three to five up two and go on forever."

4.4.2 Missing Conceptual Knowledge

This study is based on a constructivist framework and had no objective to analyze the data from a post-positivist perspective. However, there were many examples in the data where I speculated that solving the problem at hand with the use of some relevant conceptual knowledge would have made the participants' approaches more successful. For the scope of this study, I am calling missing conceptual knowledge as the knowledge that was not evident in the participants' responses. This section presents examples of missing conceptual knowledge identified from the collected data. As the examples in this section will demonstrate, this lack of knowledge was evidenced from 1) the participants' confrontation about the lack of their knowledge when they were prompted to explain their responses, or 2) the responses that suggested failure to appeal to some useful knowledge to correctly answer the given question. The list of the missing conceptual knowledge illustrated in the data is given below. The details of each along with the examples from the collected data follow.

1. Difference in the use of graphical representations of discrete and continuous impulse functions
2. Conceptual understanding of Fourier Analysis
3. Ability to translate a function from one representation to another

4.4.2.1 Difference in Use of Graphical Representations of Discrete and Continuous Impulse Functions

The data suggested that the participants lacked the knowledge of the difference in the use of the graphical representations of the discrete and continuous impulse functions.

During the interviews whenever the participants used any one type, I asked them to explain their choice to use that particular representation of the impulse function. Following are some examples from the responses received:

> I don’t know. It’s kind of hard to explain when you use them. This one’s-- supposedly goes up to infinity, and then the value given here is the area of it. And then this one has a certain height. These are used any time you have the delta function, and these are when you have a constant, I guess. (Lily)

Jake said, "I don’t know. I sometimes use this, sometimes use this one."

Rick said, "Yeah, just to show that impulse is the infinitely high, right, so just to show that, we put an arrow. And if you have just value, you put a lollipop thing."

Jim said, "These would also change to bubbles if we're doing the Fourier series, because the impulses we use would've transformed to-- and the bubbles we use with a series because the bubble is a height, but the impulse is an area."

> "I don’t-- I didn’t-- I just picked one. I didn’t actually put much thought into it. I always forget that impulses are arrows and then discreet time is circles. So I guess, in this case, it would be an arrow because these are impulses. I picked circle just kinda arbitrarily. That was-- there was no thought into that. I think it was just habit of putting a bulb" (Bill)

Paul said, "Think that's just personal preference."

Mark said, "I'm not sure. I just-- yeah, so ... I guess it was-- yeah. I don't know. I guess here, I would probably ... I'd probably do this for this one. But, yeah, I don't know."

4.4.2.2 Conceptual Understanding of Fourier Analysis

The responses of the students revealed that the participants leaned towards performing Fourier analysis using either commonly used Fourier transform pairs given in the Fourier transform table or through integration. The collected data suggested that the participants displayed lack of acumen in Fourier analysis of a signal when they were encountered with a signal, which was not easy to analyze through any of these two preferred methods. Some missing knowledge related to Fourier analysis as illustrated by the data is:

1. Ability to recognize that a function originally expressed in the form of sinusoids or exponentials is already expressed in the form of Fourier series or transform
2. Knowledge of the units of Fourier series and transform
3. Ability to identify that the Fourier transform of a signal may not exist
4. Ability to identify that Fourier series of an aperiodic signal does not exist

The description of the above-mentioned missing knowledge related to Fourier analysis and examples from the collected data are presented below.

<u>4.4.2.2.1 Ability to Recognize that a Function Expressed as Sinusoids or Exponentials is Expressed already as Fourier Series or Transform</u>

The data showed that the participants demonstrated a lack of ability to recognize that a function originally expressed in the form of sinusoids or exponentials is expressed already in the form of Fourier series or transform. A few examples demonstrating this missing conceptual knowledge are given in this section.

I asked participants to find Fourier series and transform of $v(t) = \cos\left(t + \frac{\pi}{4}\right) + 3\sin(7t)$. Luke said, "I can honestly not think of how to do the Fourier series, and so is it okay if I skip and go on?" Jake tried to solve the Fourier series through integration as shown in Figure 4.32.

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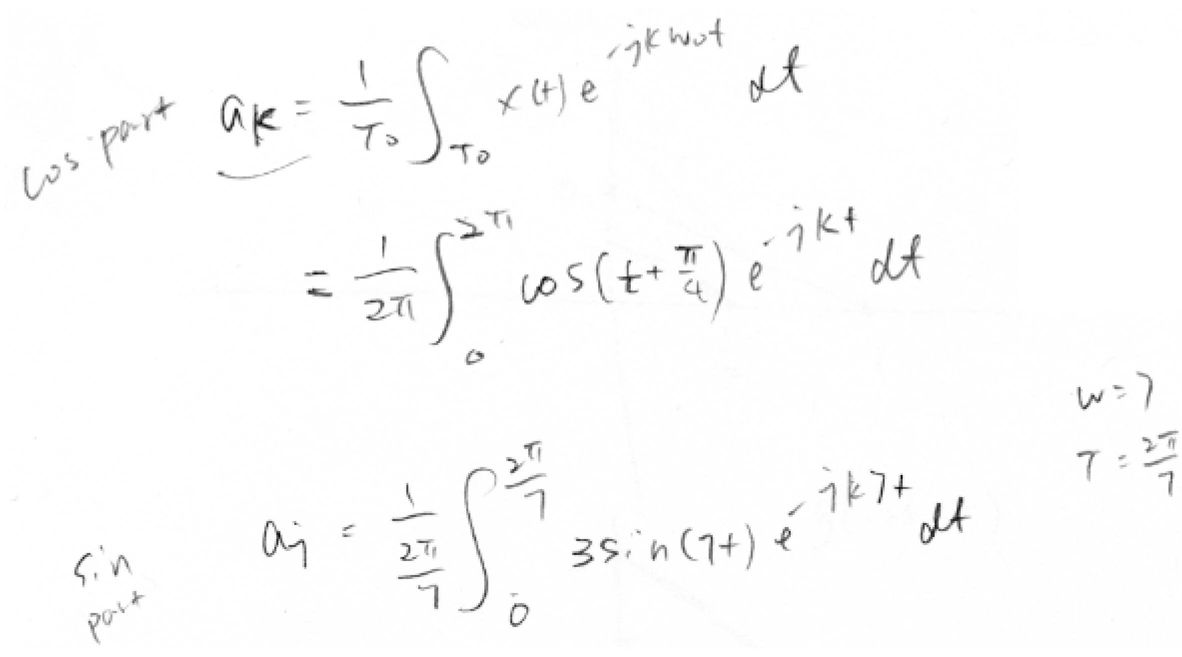

Figure 4.32. Jake's attempt to find Fourier series of $v(t) = cos\left(t + \frac{\pi}{4}\right) + 3sin(7t)$.

Megan responded in the same way as Jake's but got stuck with integrating cosine (Figure 4.33) with a phase and said, "I remember it was very annoying and it took a very long time, but I don’t really remember how to plot this anymore." Later she added, "I'm still not entirely certain how the series would relate to the transform."

$$\frac{1}{2\pi}\int_{0}^{2\pi} \cos\left(t+\pi/4\right) e^{-jk\omega_0 t}\, dt$$

Figure 4.33. Megan's attempt to find Fourier series of $v(t) = cos\left(t + \frac{\pi}{4}\right) + \ 3sin\ (7t)$.

Rick also tried to find Fourier series using integration formula as shown in Figure 4.34. He added,

> So, now, let's just consider the first part, T plus pi over 4. And it's going to be expressed as Fourier series as the sum from K negative infinity to positive infinity of K, E to JK omega naught T, so as much as a lot of impulses. But then, how do we determine A of K. A of K is one over T naught over the period X of T, E to negative JK omega naught T, right? Then, now our function is cosine, T plus pi over 4 ...

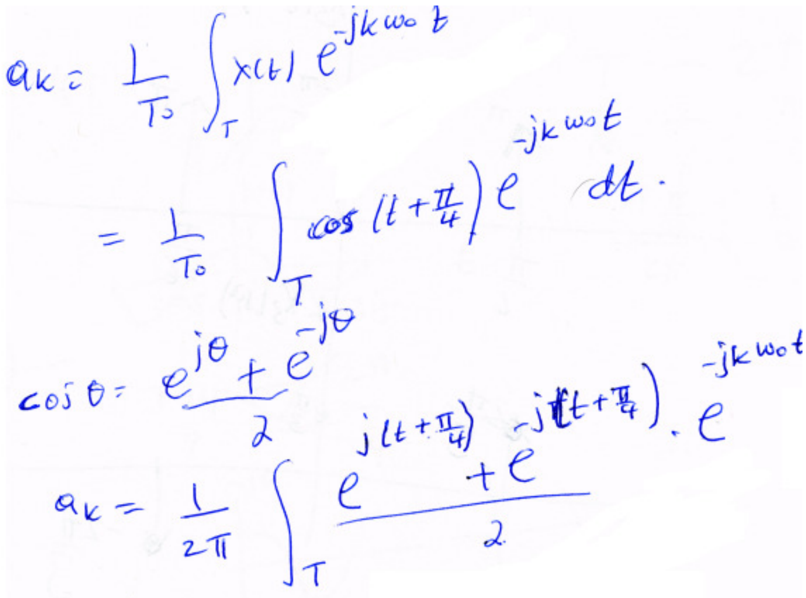

Figure 4.34. Rick's attempt to find Fourier series of $v(t) = cos\left(t + \frac{\pi}{4}\right) + 3sin\,(7t)$.

Kevin said, "I'm sorry. It's just I feel like I should have reviewed before I came here."

4.4.2.2.2 Knowledge of Units of Fourier Series and Transform

The data suggested that the participants demonstrated a lack of knowledge of the units of Fourier series and Fourier transform. If a signal v(t) has units volts, then the units of Fourier series of v(t) will be volts as well and units of Fourier transform of v(t) will be volts/Hz. A few examples demonstrating this missing knowledge are given in this section.

The participants were asked to discuss the units of Fourier series and Fourier transform of a voltage signal v(t) in volts. Following are some of the responses received:

I've always found I get my units mixed up. There's been, actually in this class too, plenty of times on the test I get the numbers right and then I don't know what units to put on it. I don't quite remember-- I know they're different. I know there's not, like, your Fourier series, I think, is just volts. And then the Fourier transform is something else. Like, it's volts square or it's something different. But to look at you and say, like, I'm sure of it? I'm not ... Because the Fourier series is still in time domain so you haven't done anything to it. It's just still-- if this is volts, then the Fourier series is volts because you're just looking at the actual equation, the v(t). But when you take the Fourier transform, you move to the frequency and I would just assume it's not the same as just volts. (Bill)

This is unit less ... Because if it has-- this is a complex number, and the units here, they cancel out, because you cannot calculate something, E to-- when you find the natural frequency, natural what is the exponent of a number. It should be unit less. (Rick)

Kevin said, "I think it's just a magnitude. So units of the magnitude as in unit less?"

For the Fourier transform I think of-- the Fourier transform of v(t) should have same units as the v(t) ... Because we know that e of j, omega-nought t, because this part is imaginary so I think it shouldn't have any unit for it ... Because I don't think we can have imaginary volts. (Jake)

> I guess if, like, taking a step back it's voltage times time. The Fourier transform goes between the time domain and the frequency domain. In the frequency domain, it doesn't have time, <laughs> because it's not in the time domain. So a unit of anything times time doesn't really make sense. Because this is probably just supposed to be a really simple question and then I move on from it. The graph of the Fourier transform shows essentially the magnitude of the signal, whatever the signal is, at that specific frequency. This being omega. That would be omega. So I guess in that sense it would make sense that it would just be volts again, because it's still just the magnitude of the signal given a specific parameter. And the same logic would go for the Fourier series as well. It would just be volts. (Ryan)

4.4.2.2.3 Ability to Identify that Fourier Transform of a Signal may not Exist

The data showed that the participants demonstrated a lack of ability to identify that Fourier transform of a signal might not exist. A few examples demonstrating this missing knowledge are given in this section.

During the interviews, nine participants were asked to explain the steps to find and plot the Fourier transform of $d(t) = t^2$. No one could identify conceptually that Fourier transform of d(t) would not exist and they all suggested they would find the Fourier transform through integration. Rick and Luke even solved the whole integration, found infinity in the answer, but still failed to reach to the right conclusion. Their responses to infinity were:

> Positive infinity ... I must have made some mistake, because it's kind of zero. Okay, if I put positive infinity, right, this would be zero. This would be also zero. This would be zero. Okay? Minus negative infinity, put zero, zero. I don't want this. It's positive infinity. You get it to positive infinity here ... I'm checking my math, if I make any mistakes. (Rick)

> But I don’t want to say that omega is just zero, but if it was zero then that makes this little term one and so then we’re looking at t-squared throughout infinite time, which kind of blows up. So I feel like there’s another way to really-- to look at it better. (Luke)

4.4.2.2.4 Ability to Identify that Fourier Series of Aperiodic Signal does not Exist

The data showed that the participants demonstrated a lack of ability to identify that Fourier series of an aperiodic signal does not exist. A few examples demonstrating this missing knowledge are given in this section.

An aperiodic signal was shown to ten participants and they were asked to explain the Fourier series of the signal. No participant was able to tell that the Fourier series would not exist. Following are some of the responses received:

> I'm thinking of like you can take your data, like certain points from that, and build up a more and more and more exact approach to it. And so, this, the Fourier transform, tries to find not, you know, the precise to the, you know, millionth decimal, but like the good, general statement of, you know, this is what, excuse

me, this is what the like first initial terms and the Fourier transform-- or the Fourier series are, sum up to. (Luke)

That's another that was like I can't remember which direction it goes but they are the same signal but off by a constant that-- I feel like it's either 2 pi or 1 over 2 pi, one of the two. But it's the same signal just scaled ... That isn't the only difference. I-- the transform is continuous and the series is discontinuous. So the scaling that I was referring to happens at whole number-- all of the points where there is a series is just they scaled value of that same point on the continuous transform. But it's-- doesn't have points for the whole transform. (Ryan)

4.4.2.3 Ability to Translate a Function from One Representation to Another

The data suggested that the participants demonstrated a lack of ability to translate a function from one representation to another. A few examples demonstrating this missing knowledge are given in this section.

When talking about the relation of the bandwidth of a rectangular function with its width in time, Justin said,

I've always had a little trouble kind of visualizing, seeing the relationship. But I think through a lot of practice, a lot of work, I think it's just ingrained. It's like a concept. So I'm not able to visually see how they're related, but maybe that's just how I think about the problems ... I forget what the time-- oh, do have this. I think it's a sinc square. Yeah. It's a sinc square. Okay. So I don't really know what a sinc square looks like, though. <laughs> I mean, I guess it's just a sinc square. I guess looking at an easier example, so I think a rectangular pulse is easier to look at.

Emily while convolving a triangular function with an impulse train said, "I like to do things graphically and see them. I think that's easier." And she could not solve graphically and said, "I misdrew that ... And then I would be stumped for a while and look at it, because I would do the Laplace transforms and do it that way and do H(S) times X(S), which would actually be just impulses." And she solved the problem correctly with the use of Laplace transform.

Ryan while explaining a problem said, "The mathematical explanation is sufficient for me to understand it so I never really looked into it more, I guess."

While trying to find Fourier transform of $x_3(t) = \sin\left(\frac{\pi}{6}t - \frac{\pi}{6}\right) + e^{j\frac{\pi}{3}}$ Jake said,

> So basically “j” is useless ... because every component occurs at different frequency. So the “j” don’t really matter here ... Because for the magnitude we will take the magnitude. So “j” is gone. We only need this part." And when asked to give an example where j will matter, he said, "It will be-- for example, if it’s j sine omega plus cosine omega ... then if we take the magnitude we need to do this, and so square each part and take the square root. In this case, the “j” matters because for this case the omega is continuous of other from negative infinity to infinity. For this case it only occurs at specific frequency.

Matt responded, "Yeah, if I can have the-- actually by math way of these two functions that would be easier for me". At a different instance, Matt also said, "...when we do the math it's really-- I think it's hard to just get the idea from the mathematical function."

Lily when finding the Fourier transform of t^2 said, "So, then, ideally I would just put that into a Maple or solving thing and then find this"

> Well, if this was an honest homework problem, I would be putting this into Maple to check it, because I'm sorry, calculus is not something I do anymore really. (Megan)

4.4.3 Mathematical Equations versus Graphs

The data showed that the students preferred to solve a problem using mathematical equations thinking that any problem is always easiest to solve using mathematical equations. Participants belonging to the CTSS-only group demonstrated this perception more often than the participants belonging to the CTSS-plus group did. In some questions, this perception led the participants to be stuck in lengthy calculations and stopped them from making conceptual decisions (outside the scope of mathematical equations) in intermediate steps that could have helped them in successfully solving that particular question. A few examples are:

Participants were asked to convolve an impulse train with a triangular function shown in Figure 4.14. A few responses are:

> If I'm going to plot y(t) I will try to transfer h(t) and x(t) to actually function so I can just use the math methods to get y(t) and plot them out and plot the y(t) ...

actually by math way of these two functions that would be easier for me ... If we can have a math to describe these two functions. (Matt)

When we were starting to learn Fourier transforms, the convolution rule was taught to us that, okay, if we had y(t) as equal to x(t) convoluted with h(t), you do that in the frequency domain and Y(ω) is going to be equal to X(ω) times H(ω). And if we can figure out the X(ω) and H(ω), it's a lot easier to multiply than to convolute, because this involves that with whatever we put in there. And this just involves multiplying two signals. And a lot of the times, if you don't have something-- if we don't have anything as complex as this where we have multiple functions based on omega, then it may be easier to revert back to the time domain with simple signals. But when I did this, I started confusing myself and talking in circles. (Tom)

Erin said, "I would probably do it mathematically and find the actual equation for what I have to use to plot that." Erin was able to solve the problem correctly through mathematical equations.

For length of a triangular function in the time domain, Emily said,

See, I never really think of it that way, to be honest. I don't look at a signal and then-- I would just do the Fourier transform, but I feel like that's been a lot more memorization and just doing problems over and over. I honestly don't ever look at a signal and go "Oh." I mean, if I saw the triangle it would be like "Oh, that's a sync squared," but I don't convert them in my head.

4.5 Summary

The findings of this study were presented in this chapter as problematic reasonings (section 4.2), mistakes (section 4.4.1), and missing conceptual knowledge (section 4.4.2). These findings are summarized in Table 4.2. The problematic reasonings in section 4.2 were categorized under three main content areas of Continuous Time Signals and Systems course content. The mistakes and missing conceptual knowledge are categorized into the same three content areas in Table 4.2.

Table 4.2. *Summary of Findings Categorized under the Three Main Content Areas*

	Problematic Reasonings	Mistakes	Missing Conceptual Knowledge
Signals Representations and Operations	1. Any property of a signal is limited within the duration of the signal itself. 2. δ(t) or δ(ω) are functions like x(t) which varies according to whatever value t takes on. (CTSS-plus) 3. The product of any function and an impulse function is a constant. (CTSS-plus)	1. Engaging with the powers of exponential functions 2. Translating a mathematical equation 3. Engaging with a unit step function 4. Engaging with an impulse function 5. Performing time shift and time scale operations combined 6. Interchange similar terms and concepts	1. Difference in the use of graphical representations of the discrete and continuous impulse functions 2. Ability to translate a function from one representation to another
Frequency Analysis	1. A periodic signal in the time domain is also periodic in the frequency domain. 2. Signal representation in the time domain is same representation in the frequency domain. 3. A constant in the frequency domain means no frequency as it has no ω in it. (CTSS-plus) 4. Phase shift means shifting the phase plot of a signal in the frequency domain. (CTSS-plus)	1. Interchange similar terms and concepts	1. Conceptual understanding of Fourier Analysis
System Analysis	1. Convolution and multiplication are interchangeable. (CTSS-only) 2. Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance. (CTSS-only)	*none seen*	*none seen*

Note.
(a) The mistake "Interchange similar terms and concepts" is placed under two content areas; Signals representations and operations, and frequency analysis because they were demonstrated in both these content areas.
(b) CTSS-only and CTSS-plus in the table represent the group that demonstrated that particular problematic reasoning more.

CHAPTER 5 - DISCUSSION

Conceptual understanding of Continuous Time Signals and Systems course content has been a challenge for undergraduate electrical engineering students despite many efforts to improve pedagogy of this course (section 2.2). The lack of the knowledge of the reasons behind the difficulties in learning the topics in this course has hindered the design of successful pedagogical methods and promotion of conceptual learning (section 2.5). The findings of this study (Chapter 4) suggest some reasons behind the difficulties in conceptually understanding Continuous Time Signals and Systems course content and the differences in the problematic reasonings of students with different academic statuses further highlights the robustness of some problematic reasonings. In my opinion, it will be important to understand the findings of this study based on potential learning challenges connected with each problematic reasoning, mistake, and missing knowledge presented in the previous chapter. The understanding of the potential learning challenges might help to understand the bigger picture of difficulties associated with the findings of this study.

Chi (2008) discusses two kinds of learning; enriching (Carey, 1991) and conceptual change (Vosniadou & Verschaffel, 2004; Chi, 2008). The enriching kind of learning occurs when a learner learns something that was missing or adds something that was incomplete in his or her prior knowledge structure. The enriching kind of learning enriches the prior knowledge structure of the learner. The awareness of the missing conceptual knowledge identified in this study is important for the design of the pedagogy of Continuous Time Signals and Systems courses to ensure enriching kind of learning of the course content. The conceptual change kind of learning occurs when the learner changes/replaces his or her prior concepts that were in conflict with the actual to-be-learned concepts that are correct by some normative standard. The knowledge of the mistakes and problematic reasonings identified in this study is important to promote the conceptual change kind of learning of Continuous Time Signals and Systems course content. The knowledge of the problematic reasonings that are present equally in the CTSS-only and CTSS-plus groups is critically important for the effective instructional design of Continuous Time Signals and Systems courses. Despite the limitations of clinical interviews (discussed in section 3.3.3.3), clinical interviews are very well recognized to explore thinking patterns of a person. For conceptual understanding related studies, the knowledge of thinking patterns gathered from interviews related to concepts that the students struggle with gives way to understanding potential underlying misconceptions (Montfort, Brown, & Findley, 2007). Therefore, the knowledge of problematic reasonings identified in this study is particularly useful because this gives an opportunity to repair thinking patterns that can potentially lead to misconceptions. The discussion of the problematic reasonings identified in this study is presented in this chapter in three parts. First, the problematic reasonings are explained with the help of possible learning challenges associated with them. In the second section, the findings of this study are compared with the difficulties in learning Continuous Time Signals and Systems courses identified in the prior literature (Chapter 2). In the last section of this chapter, implications on the instruction and learning of this course and the directions for future research are presented.

5.1 Discussion of Problematic Reasonings Identified

In the previous chapter, I grouped the problematic reasonings identified in this study as per the topics covered in the course (Table 4.1). For the sake of discussion in this chapter, I am regrouping the findings of this study according to my interpretation of the learning challenges associated with each problematic reasoning presented in Chapter 4 (section 4.2). These three learning challenges are:

LC1. Representational Translation

This learning challenge relates to the ability of a learner to translate a concept or a function from one representation to the other in the same domain (time or frequency).

LC2. Translation between Domains

This learning challenge relates to the ability of a learner to translate a concept or a function from one domain (time or frequency) to the other.

LC3. Accommodation

This learning challenge relates to the ability of a learner to alter their existing schema to learn a new concept.

This is not a unique interpretation or grouping of these findings, but it is a helpful step to start understanding the bigger picture of the learning challenges faced by the undergraduate electrical engineering students when they attempt to learn Continuous Time Signals and Systems course content. The list of the problematic reasonings (section 4.2) associated with each learning challenge is given in Table 5.1. These learning challenges along with the problematic reasonings associated with them will be discussed in detail later in this section.

Table 5.1. *Summary of All Problematic Reasonings and Possible Associated Learning Challenges*

LC1. Representational Translation	LC2. Translation Between Domains	LC3. Accommodation
LC1.1. δ(t) or δ(ω) are functions like x(t) which varies according to whatever value *t* takes on (SRO2) LC1.2. The product of any function and an impulse function is a constant (SRO3) LC1.3. A constant in the frequency domain means no frequency as it has no ω in it (FA3) LC1.4. Any property of a signal is limited within the duration of the signal itself (SRO1)	LC2.1. A periodic signal in the time domain is also periodic in the frequency domain (FA1) LC2.2. Signal representation in the time domain is same representation in the frequency domain (FA2) LC2.3. Phase shift means shifting the phase plot of a signal in the frequency domain (FA4)	LC3.1. Convolution and multiplication are interchangeable (SA1) LC3.2. Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance (SA2)

Note. SRO (signal representations and operations), FA (frequency analysis), and SA (signal analysis) are the categories in which these problematic reasonings are placed in Chapter 4 (Table 4.1)

5.1.1 Representational Translation (LC1)

The learning challenge that might be associated with the four problematic reasonings shown in Table 5.1 is the inability of the student to translate a concept or function from one representation to the other. The following learning theories might suggest an explanation to this learning challenge. As discussed in Chapter 2, conceptual understanding of an advanced mathematical operation requires a variety of simultaneously interacting mental processes like multiple visualization of the functions involved in the operation, translation of the functions into simpler and sometimes alternative forms, generalizing answer in multiple representations (Dreyfus, 1991). Piaget (1971) calls it a vertical décalage, which refers to the proficiency of understanding a concept at different stages of intellectual functioning. Furthermore, Lesh's (1981) translation model suggests that the development of deep understanding of mathematical ideas requires an ability to represent mathematical ideas in multiple ways and an ability to make connections between these multiple ways. Additionally, Redish and Smith (2008) highlight that for an engineer conceptual knowledge is the ability to not only understand mathematics (syntax) but also to combine the mathematical knowledge with the knowledge of what the math is talking about in a tightly integrated way using the meaning of the symbols.

The summary of the subset of the problematic reasonings, mistakes, and missing knowledge identified in this study that might be a result of this learning challenge are given in Table 5.2. Furthermore, this learning challenge might also help to explain why students prefer to use mathematical equations to solve the problems related to Continuous Time Signals and Systems courses (section 4.4.3). Some possible explanations of the difficulties associated with the problematic reasonings assigned to this category are given in Table 5.3.

5.1.2 Translation between Domains (LC2)

The learning challenge suggested behind the problematic reasonings assigned to this category is the inability to translate a concept from one domain to the other. The "domain" here specifically means either the time or the frequency domain. This learning challenge involves simultaneous mental processing of more than one representations of one function in two different domains. The difficult aspect of this kind of learning challenge is that almost all functions have dissimilar representations for the frequency and time domains and the process of successfully looking beyond the seemingly dissimilar representations demands advanced mental processing skills. The summary of all the problematic reasonings, mistakes, and missing knowledge identified in this study that can be explained with this learning challenge are given in Table 5.4. Additionally, the possible explanations of the learning challenges associated with the problematic reasonings assigned to this category are given in Table 5.5.

Table 5.2. *Problematic Reasonings (section 4.2), Mistakes (section 4.4.1) and Missing Knowledge (section 4.4.2) that might be Explained with Representational Translation (LC1) as a Potential Learning Challenge*

Problematic Reasonings	1. δ(t) or δ(ω) are functions like x(t) which varies according to whatever value *t* takes on 2. The product of any function and an impulse function is a constant 3. A constant in the frequency domain means no frequency as it has no ω in it 4. Any property of a signal is limited within the duration of the signal itself
Mistakes	1. Engaging with the powers of exponential functions 2. Translating a mathematical equation 3. Engaging with a unit step function 4. Engaging with an impulse function 5. Performing time shift and time scale operations combined
Missing Conceptual Knowledge	1. Difference in the use of graphical representations of discrete and continuous impulse functions 2. Conceptual understanding of Fourier Analysis a. Ability to recognize that a function originally expressed in the form of sinusoids or exponentials is already expressed in the form of Fourier series or transform 3. Ability to translate a function from one representation to another

Table 5.3. *Explanation of Problematic Reasonings with Representational Translation (LC1) as a Potential Learning Challenge*

Problematic Reasoning	Explanation
LC1.1. $\delta(t)$ or $\delta(\omega)$ are functions like $x(t)$ which varies according to whatever value *t* takes on	Despite exhibiting an understanding of the shape and nature of an impulse function, the participants interchanged the symbol $\delta(t)$ with the symbol $x(t)$ and failed to translate the symbol $\delta(t)$ into its correct graphical representation
LC1.2. The product of any function and an impulse function is a constant	The product of any function with an impulse function requires simultaneous interaction of mental processes including graphical representation of the impulse function, the function being multiplied, process of multiplication, translation of graphs into numbers for multiplication and then translating numbers into graphical form as a product. Despite showing sufficient understanding of the nature and shape of an impulse function, the participants failed to translate the product of a function with an impulse function to its graphical and symbolic representation
LC1.3. A constant in the frequency domain means no frequency as it has no ω in it.	Participants exhibited failure in translating a constant number from numerical representation to verbal as well as graphical representation
LC1.4. Any property of a signal is limited within the duration of the signal itself.	Participants seemed to fail to translate the graphical representation of a signal to the graphical representation of either properties of that signal or a new signal obtained by performing some operation on that signal.

Table 5.4. *Problematic Reasonings (section 4.2), Mistakes (section 4.4.1) and Missing Conceptual Knowledge (section 4.4.2) that might be Explained with Translation Between Domains (LC2) as a Potential Learning Challenge*

Problematic Reasonings	1. A periodic signal in the time domain is also periodic in the frequency domain 2. Signal representation in the time domain is same representation in the frequency domain 3. Phase shift means shifting the phase plot of a signal in the frequency domain
Mistakes	None of the mistakes can be explained best with LC2
Missing Conceptual Knowledge	1. Conceptual understanding of Fourier Analysis a. Knowledge of the units of Fourier series and transform b. Ability to identify that the Fourier transform of a signal may not exist c. Ability to identify that Fourier series of an aperiodic signal does not exist

Table 5.5. *Explanation of Problematic Reasonings with Translation Between Domains (LC2) as a Potential Learning Challenge*

Problematic Reasonings	Explanation
LC2.1. A periodic signal in the time domain is also periodic in the frequency domain	When translating a periodic signal from one domain to the other, the participants seem to fail to mentally let go of the periodic nature of the signal and fail to open their minds to the possibilities of aperiodic nature of its properties.
LC2.2. Signal representation in the time domain is same representation in the frequency domain	When translating a signal from one domain to the other, the participants seem to fail to mentally let go of the signal representation they encounter first.
LC2.3. Phase shift means shifting the phase plot of a signal in the frequency domain	Participants seem to interchange a time domain concept with a frequency domain concept and fail to translate the shift operation in one domain to its consequent operation in the other domain

5.1.3 Accommodation (LC3)

The problematic reasonings assigned to this category seem to come from the challenge of accommodation of the newly learned knowledge. Accommodation is the term introduced by Piaget (1971) which refers to the adaptation process of a learner while altering an existing schema (cognitive framework or concept that helps organize and interpret information) because of the newly learned information. For example, a child may have an existing schema for table. Table has four legs, so the child may automatically believe that any furniture item with four legs is a table. When the child learns that chair also have four legs, he/she will undergo a process of accommodation in which his/her existing schema for tables will change and he/she will also develop a new schema for chairs. Accommodation is specifically challenging when the new information conflicts with the existing schemas.

The summary of the problematic reasonings, mistakes, and missing knowledge identified in this study that can be explained with this learning challenge are given in Table 5.6. Additionally, some possible explanations of the learning challenges associated with the problematic reasonings assigned to this category are given in Table 5.7.

Table 5.6. *Problematic Reasonings (section 4.2), Mistakes (section 4.4.1) and Missing Conceptual Knowledge (section 4.4.2) that might be Explained with Accommodation (LC3) as a Potential Learning Challenge*

Problematic Reasonings	1. Convolution and multiplication are interchangeable 2. Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance
Mistakes	1. Interchange similar terms and concepts
Missing Knowledge	None of the missing knowledge can be explained best with LC3

Table 5.7. *Explanation of Problematic Reasonings with Accommodation (LC3) as a Potential Learning Challenge*

Problematic Reasoning	Explanation
LC3.1. Convolution and multiplication are interchangeable	Students might find difficult to accommodate the concept of convolution in their existing schema of multiplication because 1. The process of convolution involves multiplication, and 2. The convolution of two signals in one domain corresponds to multiplication in the other domain
LC3.2. Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance	The participants seem to fail to accommodate the concept of time-invariance of a systems in their existing schema of the literal translation of time-invariance

5.2 Discussion of Differences in Problematic Reasonings of Students with Different Academic Statuses

The comparison of the difference of the use of the problematic reasonings identified in this study among the CTSS-only and CTSS-plus groups (Figures 4.16 and 4.17) revealed that students with different academic statuses draw on different problematic reasonings. For the sake of discussion, the problematic reasonings that are used more often by the CTSS-only group are called as CTSS-only-dominant problematic reasonings, the problematic reasonings that are used more often by the CTSS-plus group are called as CTSS-plus-dominant problematic reasonings, and the problematic reasonings employed almost equally by both groups are called as robust problematic reasonings. This section discusses the differences and the possible reasons behind the differences in problematic reasonings employed by students with different academic statuses.

5.2.1 CTSS-Only-Dominant Problematic Reasonings

The problematic reasonings that are employed more often by the participants belonging to the CTSS-only group are:

1) Convolution and multiplication are interchangeable (LC3.1)
2) Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance (LC3.1)

Both these problematic reasonings are described in section 5.1 as related to the accommodation of the newly introduced concepts. The fact that these problematic reasonings are not prevalent in the CTSS-plus group implies that the students reach equilibration stage of these concepts as they continue to use the content of Continuous Time Signals and Systems courses in advanced courses. Such problematic reasonings are easy to repair and may only require more practice on students' behalf to overcome this learning challenge. There is also evidence in the data collected from the participants of the CTSS-plus group that repetition and practice helped the participants to overcome these learning challenges. For example, Tom, who had taken three courses that require prior knowledge of Continuous Time Signals and Systems course content (Table 3.2), said about convolution,

> From, so, this is a 123[1] concept, which is dynamical systems of convolution. That's when we were first introduced to that, and I kind of struggled with grasping that concept. But it made more sense to me in discrete time, which is why I mentioned that earlier ... I guess repetition makes me understand it. (Tom)

5.2.2 CTSS-Plus-Dominant Problematic Reasonings

The problematic reasonings employed more often by the participants belonging to the CTSS-plus group are:

1) $\delta(t)$ or $\delta(\omega)$ are functions like x(t) which varies according to whatever value *t* takes on (LC1.1)
2) The product of any function and an impulse function is a constant (LC1.2)
3) A constant in the frequency domain means no frequency as it has no ω in it (LC1.3)
4) Phase shift means shifting the phase plot of a signal in the frequency domain (LC2.3)

Most of the problematic reasonings used dominantly by the CTSS-plus group belong to the learning challenge of representational translation. There can be numerous possible explanations for these problematic reasonings prevalent only in students from the CTSS-plus group. It seems counter-intuitive but studies have shown that students can get more confused after more coursework on a concept (Chi & Roscoe, 2002; Limón & Mason, 2002; Mayer, 2002; Montfort, Brown, & Pollock, 2009). Conceptual change requires revision of beliefs and when students try to reevaluate their knowledge of the

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[1] Course code is replaced by 123 to protect privacy of the institution

subject matter, their confidence in their knowledge decreases as they start questioning the facts on which they based their foundational knowledge. Additionally, recently learned concepts can inhibit the recollection of previously learned concepts, a phenomenon called retroactive interference (Wohldmann, Healy, & Bourne, 2008).

Exact reasons for the presence of these problematic reasonings in this course's content can be explored by conducting a longitudinal study of how conceptual understanding of undergraduate electrical engineering students evolve as they progress through the electrical engineering curriculum. However, there is some evidence in the collected data that speaks for the presence of retroactive interference of the concepts that students learned after taking Continuous Time Signals and Systems course. A few examples are:

> Fourier transform has integration but Fourier series has summation and while I am thinking of summation I started thinking about equations with summations in them and I was thinking x of lambda h of t minus lambda which is definitely discrete time convolution and totally not what we are doing. (Ryan)

Additionally, when Ryan was explaining the units of Fourier series and Fourier transform, he and I had this conversation:

> Ryan: The only thing that comes to mind for both of these is volts squared.
>
> Me: Volts squared?
>
> Ryan: Mm-hm.
>
> Me: And why is that?
>
> Ryan: Because of a lab I had for a different class ...

Carl when talking about finding integral of a periodic signal using superposition said,

That's not superposition, is it? Because I cannot remember for the life of me what that is. I remember using it in another class. I'll come back to that one here in a little bit. I'll just make a note. (Carl)

Furthermore, there is evidence in the interview data (mostly from the members of the CTSS-plus group) collected for the study that students are relying on computers for even solving trivial problems in this course. The undue use of computer programs to reach to a conclusion readily might be a reason for the lack of the ability to translate a signal from one representation to the other and might be hindering the development of metacognition skills of the students. For Continuous Time Signals and Systems courses at Iris University, students are not allowed to use Maple or similar programs for problem solving and home works are not accepted in the form of Maple worksheets. The fact that the students are still using Maple when they get out of the class further suggests a resistance to learning this material on the behalf of students. I will present some evidence of the students' preference of the use of computers over reasoning to solve a problem from the collected data here:

While trying to find Fourier series of $v(t) = \cos\left(t + \frac{\pi}{4}\right) + 3\sin(7t)$ Bill said

To be honest, at this point, I’d be plugging this into a computer program to let them do that for me and help out. (Bill)

> Well, if this was an honest homework problem, I would be putting this into Maple to check it, because I'm sorry, calculus is not something I do anymore really. But I think you can calculate that one by doing integral of that times that plus the integral of that, sine times that. I'm thinking derivatives, aren't I? Anyway, the point is, put this into Maple, I'm going to get something that is going to be a sine. (Megan)

Additionally, Luke while trying to convolve an impulse train with a rectangle said, "I would probably use my experience and MATLAB." Furthermore, while trying to find Fourier transform of t^2, Lily said, "So, then, ideally I would just put that into a Maple or solving thing and then find this."

5.2.3 Robust Problematic Reasonings

The problematic reasonings that are exhibited almost equally by the participants in both groups are:

1) Any property of a signal is limited within the duration of the signal itself (LC1.4)
2) A periodic signal in the time domain is also periodic in the frequency domain (LC2.1)
3) Signal representation in the time domain is same representation in the frequency domain (LC2.2)

The word "robust" for these problematic reasonings is borrowed from the term "robust misconceptions." The term robust misconception is used in thousands of conceptual change studies starting from Novak (1977) until today (Richey & Nokes-Malach, 2014). Robust misconceptions are conceptions that are incompatible with the standard scientific conceptions and are difficult to revise which makes a conceptual change hard to achieve (Chi, 2008). Therefore, "radical conceptual change" is required to build conceptual understanding in the presence of robust misconceptions (Carey, 1985); a term used in the literature to emphasize the revision of overall belief system. On the same lines, these problematic reasonings are problematic because these are incompatible with the standard correct reasonings. Moreover, the fact that these problematic reasonings are present almost equally in students from both groups speaks for the difficulty to revise these problematic reasonings, hence robustness.

Of the three robust problematic reasonings, two might be explained with the learning challenge of translation between domains. The ability to successfully translate between domains requires multiple aspects of mental processing at the same time, which includes simultaneous processing of two or more entirely different representations of the same signal, placing them in two entirely different domains, and answering related questions all at the same time. The acquisition of this skill demands careful attention to instructional methods as this kind of skill falls under the highest levels of revised Bloom's taxonomy. Carefully designed hands-on application-oriented activities that provide personal experiences with the Continuous Time Signals and Systems courses along with related prompts for metacognition (Simoni, Fayyaz, & Streveler, 2014) seem like a good pedagogical strategy for repairing these robust problematic reasonings.

5.3 Discussion of Problems in Learning Continuous Time Signals and Systems Courses in Light of Problematic Reasonings

There are a few quantitative studies conducted in the past to identify the problems in learning Continuous Time Signals and Systems course content among undergraduate electrical engineering students. The motivation of this study was to understand the reasons behind the problems in learning this course. In this section, I will discuss how the findings of this study might suggest an explanation for the reasons behind some of the previously identified problems in learning content of this course.

Previous studies on learning Continuous Time Signals and Systems courses have shown that students face difficulty in connecting dissimilar shapes of the same signal in different domains (Wage, Buck, & Wright, 2004; Buck & Wage, 2005; Wage, Buck, & Hjalmarson, 2006a). Of the nine problematic reasonings identified in this study (Table 5.1), seven relate to the translation of a function and three of the seven specifically relate to the translation between domains. These problematic reasonings extend some possible explanations for the difficulties faced by the students when engaging with the problems related to connecting dissimilar shapes of the same signal in different domains.

Moreover, studies in the past have contended that Continuous Time Signals and Systems course content are difficult to learn conceptually because of the extensive use of the mathematical modeling and formulas involved in this course (Nasr, Hall, & Garik, 2005; Ferri et al., 2009; Han, Zhang, & Qin, 2011; Tsakalis et al., 2011). The findings of this study suggest that the poor grades in this course might not be due to extensive mathematics or students' lack of mathematics proficiency, but due to students' inability to process at the highest level of Bloom's taxonomy of learning which requires a simultaneous processing of a combination of mathematical concepts and Continuous Time Signals and Systems related concepts. Additionally this also suggests an explanation for the results of the correlation of students' grades in the pre-requisite mathematics courses and Continuous Time Signals and Systems courses, which showed that even the students with grades of As and Bs in the pre-requisite mathematics courses could not perform well in Continuous Time Signals and Systems courses (Simoni, Fayyaz, & Streveler, 2014).

Many previous studies contend that Continuous Time Signals and Systems courses are difficult to learn because large part of these courses deals with abstract mathematical concepts that are difficult to visualize and hard to make sense of (Shaffer, Hamaker, & Picone, 1998; Nasr, Hall, & Garik, 2005, 2007; Tsakalis et al., 2011). The seven problematic reasonings (Table 5.1) identified in the study that involve translation of the signal might suggest an explanation for the difficulty in visualization of some of the mathematical concepts in these courses.

Furthermore, as discussed in Chapter 2, past studies have suggested that students face difficulties in performing convolution by graphical method and these difficulties might be explained with p-prims of the students (Nasr, Hall, & Garik, 2007, 2009). Two problematic reasonings (a. interchange of multiplication and convolution and, b. any property of a signal is limited within the property of the signal itself) identified in this study might extend further explanation for Nasr, Hall, and Garik's (2007, 2009) justification behind students' difficulties in performing convolution by graphical method. In addition, Wage, Buck, and Wright (2004) claimed that students think that multiplication in the time domain is multiplication in the frequency domain as well

(Wage, Buck, & Wright, 2004). Two problematic reasonings from this study (a. Students' attempt to interchange multiplication and convolution, and b. Signal's representation is the same in both the time and frequency domains) might extend an explanation for why students would think that multiplication in the time domain is multiplication in the frequency domain.

In addition, as discussed in Chapter 2, Nelson, Hjalmarson, and Wage (2011) observed that students exhibited significant gaps in their knowledge of i) definitions and/or evaluation of the conditions of causality and stability of a system, ii) mathematical representation of signals and systems as either a function or a graph, iii) working with different types of independent and dependent variables together in a function, and v) understanding a system in terms of a signal (impulse response). Except for the gap in the knowledge of causality and stability of a system because the protocol used for the study had no related questions, the problematic reasonings identified in this study suggest explanations for the gaps in the knowledge identified by Nelson, Hjalmarson, and Wage (2011).

Table 5.8 shows a summary of the concepts involved in the consistently incorrectly answered questions in SSCI tests obtained from the analysis of scores of the students (Table 2.1) along with the possible difficulties suggested from the findings of this study.

Table 5.8. *Possible Explanations Suggested by the Findings for Incorrectly Answered Questions in SSCI Tests (section 2.4.2)*

Concept Covered in the Incorrectly Answered Question	Problematic Reasonings that might Account for the Incorrect Response
Multiplication in the time domain	Convolution and multiplication are interchangeable (LC3.1)
Convolution by graphical method	Convolution and multiplication are interchangeable (LC3.1)
Time Invariance	Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance (LC3.2)
Fourier transform at ω=0 and area under the time domain signal	Signal representation in the time domain is same representation in the frequency domain*
Time flip and shift operations combined	Evidence available in the mistakes of students presented in section 4.4.1.5
LTI causality	_**

Note.

* It was noted that the most incorrect response was that students were thinking that the value of F(ω) for ω=0 is the value of f(t) at t=0 which aligns with the given problematic reasoning

** There was no question in the protocol that prompted participants to talk about this concept

5.4 Why Continuous Time Signals and Systems Courses are Difficult

Continuous Time Signals and Systems courses are considered particularly hard among other courses in the electrical engineering curriculum. This is evident from the results of the quantitative analysis of withdrawal and failure rates of almost 860 undergraduate electrical engineering students over a period of ten years, conducted by Simoni, Fayyaz, and Streveler (2014). The results showed that the withdrawal and failure rates in Continuous Time Signals and Systems and Electromagnetics courses were approximately three times higher than in other required courses in electrical and computer engineering curriculum at Rose-Hulman Institute of Technology.

The problematic reasonings identified in this study suggest explanations for students' struggles in understanding this course. I will use the knowledge of the problematic reasonings gained from this study and the course content map shown in Figure 5.1 to illustrate why this course might be so hard. I made this map to design a Continuous Time Signals and Systems course for a class project. In Figure 5.1, "Important to know" topics are the ones without conceptual understanding of which the proficiency of the students in this course would be incomplete. Furthermore, "enduring understandings" are the topics that the students should get the thorough understanding of and retain even after they have forgotten many other details covered in this course (Wiggins & McTighe, 2005). To keep the discussion focused on the conceptual learning of basic concepts, "worth-being-familiar-with" topics in Continuous Time Signals and Systems course content are not shown in Figure 5.1.

An important observation in the complete map of the topics covered in typical Continuous Time Signals and Systems courses is that the "important to know" topics are suggested to be well-understood for more than one "enduring understandings" in this course, and in some cases these "important to know" topics are suggested to be well-understood for all three "enduring understandings". For example, good understanding of "mathematical representations of signals" is deemed important for conceptual learning of all four enduring understandings. This recommends that failure to understand these "important to know" topics can cripple the conceptual learning of enduring understandings of this course.

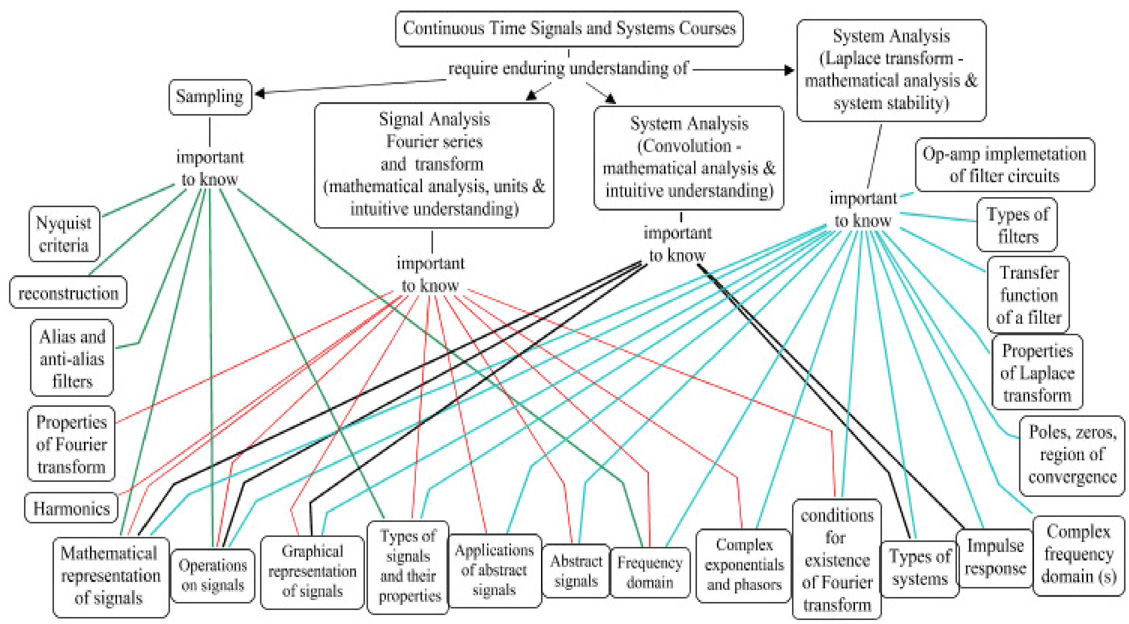

Figure 5.1. Map of Curricular priorities of typical Continuous Time Signals and Systems courses. Note that topics that are marked 'Important to know' are all the concepts that help students learn enduring understandings.

Figures 5.2, 5.3, and 5.4 show mappings of the topics within Continuous Time Signals and Systems course content whose conceptual learning might be directly affected from the use of problematic reasonings identified in this study. These figures extend an explanation for how the use of the problematic reasonings identified in this study might hinder the conceptual understanding of Fourier analysis, convolution, and Laplace transform. A few common observations from Figures 5.1, 5.2, and 5.3 are:

1. Even though there was no direct question in the protocol used for this study about Laplace transform (Table 3.1), yet the problematic reasonings identified in the study present a potential barrier in the conceptual learning of Laplace transform too.
2. Conceptual understanding of representations of signals through mathematical equations and graphs is important-to-know for all four enduring understandings in this course. However, all the robust and CTSS-plus-dominant problematic reasonings present a potential to create difficulty in the conceptual understanding of representations of signals using mathematical equations and graphs.

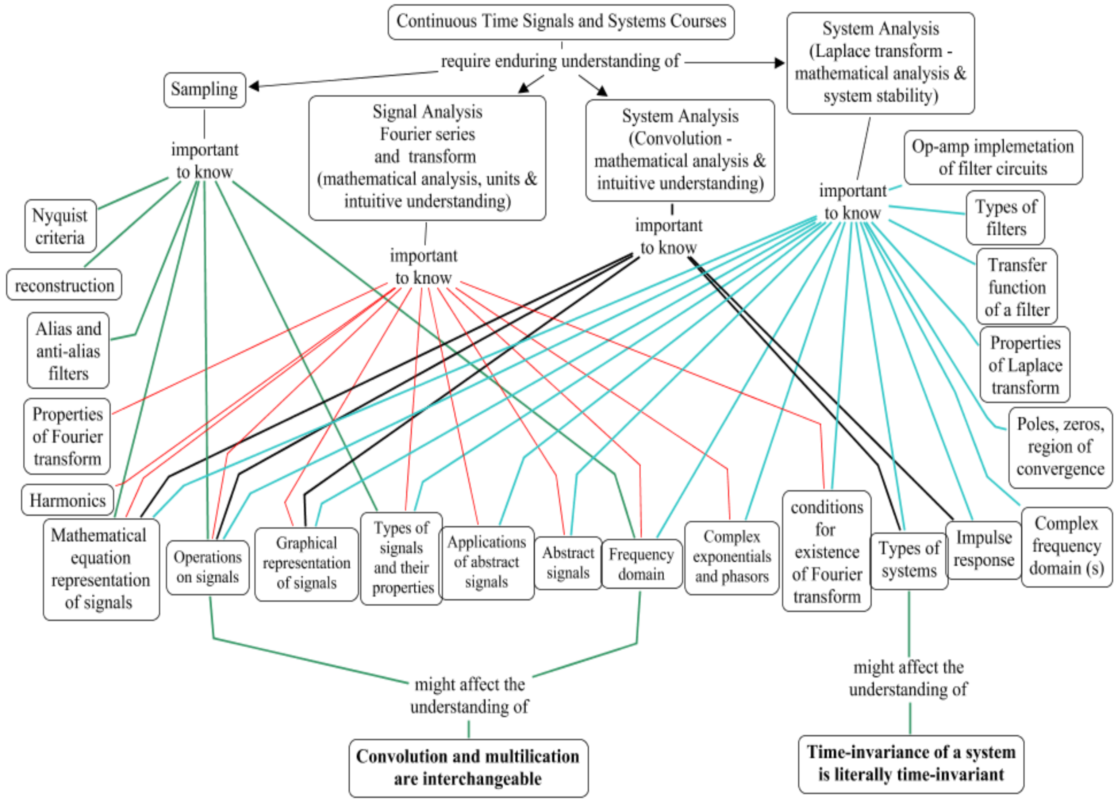

Figure 5.2. Topics of Continuous Time Signals and Systems course content and CTSS-only-dominant problematic reasonings. This map shows when conceptual learning might be affected by these problematic reasonings.

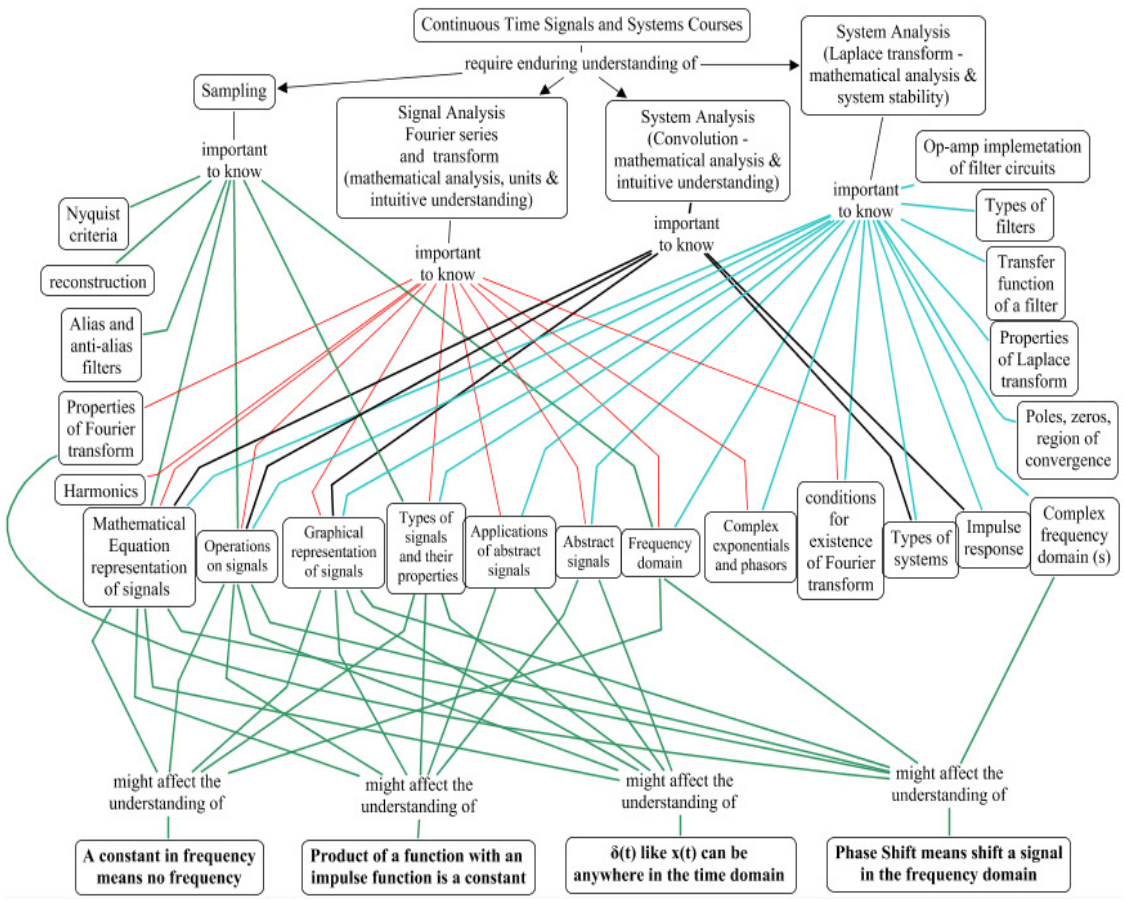

Figure 5.3. Topics of Continuous Time Signals and Systems course content and CTSS-plus-dominant problematic reasonings. This map shows when conceptual learning might be affected by these problematic reasonings.

3. Figure 5.2 shows that CTSS-only-dominant reasonings do not demonstrate a potential to affect understanding of many topics together. This might be a reason they do not persist and are easy to fix.
4. Figure 5.4 shows that the robust problematic reasonings illustrate a potential to affect the understanding of maximum number of topics in this course. Their widespread prevalence suggests an explanation for them being robust.

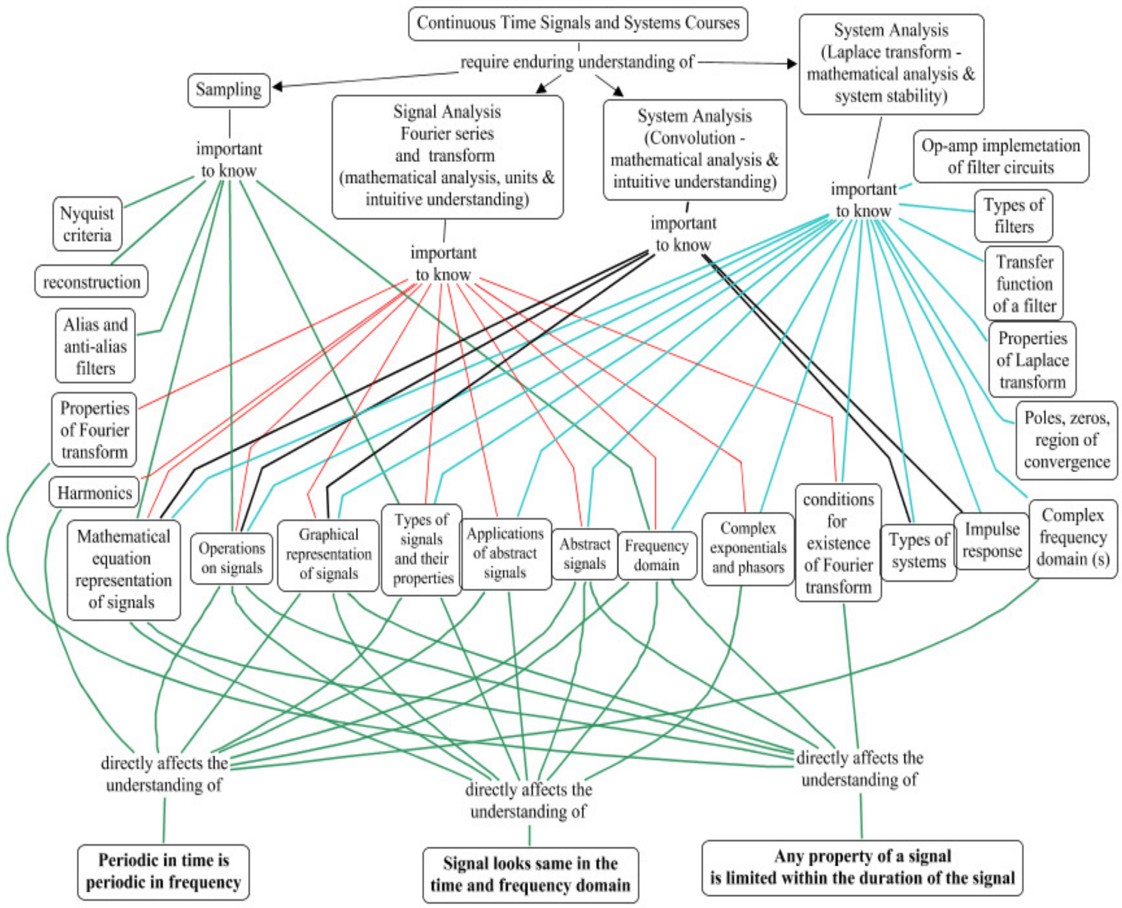

Figure 5.4. Topics of Continuous Time Signals and Systems course content and robust problematic reasonings. This map shows when conceptual learning might be affected by these problematic reasonings.

Table 5.9 provides a summary of the information in Figures 5.2, 5.3, and 5.4 about each problematic reasoning identified in this study and the conceptual understanding of important-to-know content areas in Continuous Time Signals and Systems courses that can be affected by that particular problematic reasoning.

Table 5.9. *Important-to-Know Content Areas in Continuous-Time Signals and Systems Courses where Conceptual Understanding can be Affected by the Identified Problematic Reasonings*

Problematic Reasonings		Important to know (IK) Concepts Affected
CTSS-Only Dominant	Convolution and multiplication are interchangeable (SA1)	IK-2, IK-7
	Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance (SA2)	IK-13
CTSS-Plus Dominant	δ(t) or δ(ω) are functions like x(t) which varies according to whatever value t takes on (SRO2)	IK-1, IK-2, IK-3, IK-4, IK-5, IK-6
	The product of any function and an impulse function is a constant (SRO3)	IK-1, IK-2, IK-3, IK-4, IK-5, IK-6
	A constant in the frequency domain means no frequency as it has no ω in it (FA3)	IK-1, IK-2, IK-3, IK-4, IK-7
	Phase shift means shifting the phase plot of a signal in the frequency domain (FA4)	IK-1, IK-2, IK-3, IK-7, IK-8, IK-12
Problematic Reasonings Present Almost Equally in both Groups	Any property of a signal is limited within the duration of the signal itself (SRO1)	IK-1, IK-2, IK-3, IK-5, IK-7, IK-11, IK-12
	A periodic signal in the time domain is also periodic in the frequency domain (FA1)	IK-2, IK-3, IK-4, IK-5, IK-6, IK-7, IK-8, IK-10
	Signal representation in the time domain is the same representation in the frequency domain (FA2)	IK-1, IK-2, IK-3, IK-4, IK-5, IK-6, IK-7, IK-8

Note. Identification codes for Important to Know (IK) concepts from the concept maps shown in Figures 5.1, 5.2, 5.3, and 5.4 are: Mathematical equation representation of signals (IK-1), Graphical representation of signals (IK-2), Operations on signals (IK-3), Types of signals and their properties (IK-4), Abstract signals (IK-5), Application of abstract signals (IK-6), Frequency domain (IK-7), Complex frequency domain (IK-8), Complex exponentials and phasors (IK-9), Harmonics (IK-10), Conditions for existence of Fourier transform (IK-11), Properties of Fourier transform (IK-12), Types of systems (IK-13), and Impulse response (IK-14)

5.5 Implications

Learning Continuous Time Signals and Systems course content is challenging. Many topics in the course content have multiple representations and all the multiple representations are used interchangeably. Moreover, many concepts are abstract which adds to the challenge of learning. Additionally, conceptually learning only one representation of a concept that has more than one representation does not solve the problem of learning as the use of different representations help in different problems. This in turn influences the confidence of the learners as well, as they stay confused and continue to question their understanding every time they encounter a seemingly well-learned (in one isolated situation) concept in a new situation. Furthermore, as demonstrated in the findings of this study, the problematic reasonings used by the students when they access content of Continuous Time Signals and Systems courses are intricately intertwined with multiple topics in these courses, which further begs for conscious teaching and learning of the topics covered in these courses. Following are my suggestions to facilitate conceptual learning and teaching of these courses.

5.5.1 Implications for Instruction

1. Learning of Continuous Time Signals and Systems courses will improve if the instructors make sure that the students understand where each individual course content fits in the bigger picture (enduring understandings, Figure 5.1) of the whole course and in the whole electrical engineering curricula. For example, when I taught this course, I used to make little concept maps on the white board in the classroom both at the start of the course and at the intermediate stages during the semester

whenever I introduced a new concept to help my students understand the bigger picture (Fayyaz, 2009). For instance, I used to teach Laplace transform after Fourier transform and at the beginning of the introduction of Laplace transform; I used to make a content map in the class involving my students to connect Laplace transform with the course content already covered at that point. This also helped to give a heads up to the students about the differences and similarities between Fourier and Laplace transforms through the concepts already covered or to-be covered in the course. Doing the same exercise in the class for the next topic, which in my case used to be application of Laplace transform in system analysis, the previous conceptual connections were reiterated and misunderstandings were discussed before the start of a new topic. In my opinion, helping the students to make connections between the topics covered in the whole course and later in the courses that require prior knowledge of Continuous Time Signals and Systems course content empowers students for conceptual learning of the course content and "makes the game worth playing" for them (Perkins, 2010).

2. There should be labs in this course that are well-aligned with the course content to further nudge students' understanding of the course content. One such example is the lab instruction model at the Rose-Hulman Institute of Technology for this course (Simoni, Aburdene, & Fayyaz, 2013b, 2013c, 2013d, 2014). At Rose-Hulman Institute of Technology, the instructor of the course conducts the labs himself making sure that the concepts covered in the classroom are highlighted in the labs and same terms are used in the labs as are discussed in the classroom. I contend that this helps to rectify the problem of interchanging terms suggested in this study. Additionally, at

Rose-Hulman Institute of Technology, hands-on application-oriented activities are designed for lab work that provide personal experiences to students with signals like their own ECG, speech, and hearing. The experience of applying new knowledge (Garfield, 1995), engaging in solving real-world problems (Merrill, 2002), and knowing that new knowledge is useful in the daily life (Çetin, 2004) promote conceptual learning. Moreover, reflection prompts are given to the students in each lab session that encourage them to connect theoretical knowledge covered in the classroom with their real-world personalized experiences in the labs. These reflections are graded which further pushes the students to reflect and learn. For learning mathematics related topics, reflection empowers learners to separate themselves from the action of doing mathematics and think on the processes under study (Wheatley, 1992). I argue that lab activities with real life applications aligned with the course content and reflections help to rectify the difficulty faced by the students in translation of the signals suggested in this study. Furthermore, studies have shown, in general, bridging the gap between theory-focused lectures and application-oriented expectations of undergraduate engineering students increase the motivation of a student (Munz, Schumm, Wiesebrock, & Allgower, 2007).

3. Instructors need to give careful attention to the hard parts (Perkins, 2010) of these courses. The hard parts of these courses include, i) multiple representations of the same function, ii) multiple approaches available to solve a question, iii) different skill sets needed to solve a question after making small changes in the question. Instructors need to give examples of different representations and skill sets in the class as much as possible to make the minds of the students engaged in all possible situations.

Studies have shown that the students learn better and are more motivated to struggle with their learning when they work cooperatively in small groups to solve problems and learn to argue convincingly for their approach among conflicting ideas and methods (Garfield, 1995). I argue that for teaching Continuous Time Signals and Systems courses, facilitating group discussions in the classroom would help to explore multiple representations and methods for any given situation as different students will attempt to look at the given question differently which will provide opportunities for other students to open their minds to the possibilities of multiple solutions and representations. Additionally, class discussions will help to pinpoint and fix problems in the reasonings employed by the students when they engage with the course content. Furthermore, students always find difficulty in understanding the concepts that they cannot understand intuitively from the mathematical expression or vice versa (Bruner, 1962; Ashcraft, 2002). Facilitating students to reflect intuitively (through well-designed homework, class discussions, etc.) on the concepts taught in these courses will help them better understand the course content.

4. I recently learned about the use of Licht's model of teaching electrical energy, voltage, and current (Licht, 1991) for teaching related courses like ac circuits within undergraduate electrical engineering curriculum (Pitterson & Streveler, 2014). This model provides an alternative framework to effectively teach courses with abstract and complex scientific concepts like alternating current. I argue that this model seems effective for teaching Continuous Time Signals and Systems courses because of the abstract nature of the topics covered in this course. This model divides instruction of a concept or a topic into five stages of instruction. I will discuss these stages briefly to

explain the instruction model and I will use the example of teaching Fourier transform to explain how this model might work for the overall instruction of Continuous Time Signals and Systems courses.

Stage 1 is a phenomenological orientation of the concept. This includes introduction of the general overview of the concept and an overall intersection of the macro, micro, and symbolic forms of that concept. For teaching the concept of the Fourier transform at stage 1, the instructor will introduce the term, its definition, and possible relations of this newly introduced term with the previous knowledge of the students. This stage provides students to discuss and exchange ideas on their beliefs on signals and its frequency components, which will prepare students for a more in-depth explanation of the topic. At this stage the instructor would make sure that the students' understandings about the signal consisting of different frequency components is clearly established.

Stage 2 introduces the qualitative macroscopic approach to the concept. This includes introduction of more specific information about the particular concept but without any micro level details. This stage helps students to connect their prior knowledge with the newly introduced knowledge. For teaching the concept of the Fourier transform at stage 2, the instructor at this stage would introduce the terms like complex exponentials and Eigenvectors at the macro level. This stage would enable students to understand the bigger picture of Fourier transform and the related terms.

Stage 3 introduces a qualitative microscopic approach. This includes introduction of the microscopic details of the concept expecting that the students at this stage are ready to switch between macroscopic and microscopic details of that concept. For teaching the concept of the Fourier transform at stage 3, the instructor would use visual representations and simulations to facilitate students to assimilate the abstract concept of frequency domain and frequency. This is very useful because students do not have a prior concept of frequency domain at this point and understanding the representations help to understand the importance of this concept.

Stage 4 introduces the quantitative macroscopic approach of the new concept. This means introduce mathematical equations and formulas related to the new concept at this stage. For teaching the concept of the Fourier transform at stage 4, the instructor would introduce the mathematical formulas of Fourier transform at this stage after they are sure that the students have sufficient qualitative understanding of the newly introduced concept at both macroscopic and microscopic levels.

Stage 5 introduces a quantitative microscopic approach. This includes providing students with opportunities to explore and verify the relationships between various mathematical symbols and variables introduced in the previous stage and the details of the concept itself. For teaching the concept of the Fourier transform at stage 5, the instructor would give multiple problems to students to facilitate them to explore and verify the relationship of signals and their frequencies with mathematical equations. In order to ensure students completely understand the fundamental underlying relationship among signals, Fourier transform formulas, and frequencies, the instruction at this stage would highlight the microscopic level of equations enabling students to fully understand how variables are related as well develop the ability to derive equations from textual information. At this stage, the instruction on the use of mathematical equations and quantitative approaches to learning Fourier transform should go beyond the rote learning and application of the formulas. At this stage, students must be prepared to understand when a formula or approach is more appropriate to use and what relationship each formula represents.

5.5.2 Implications for Learning

Garfield (1995) argued that students learn better if they are engaged in, and motivated to struggle with their own learning. He asserted that working cooperatively in small groups to solve problems and learning to argue convincingly for their approach among conflicting ideas and methods is helpful for students (Garfield, 1995). For learning topics such as those in Continuous Time Signals and Systems course content, this seems like a good strategy because this might help students to reflect on their choices for problem-solving strategies when choosing among multiple interchangeable and conflicting methods and signal representations presented in these courses.

Additionally, for learning abstract concepts that involve mathematics, it is important for the learners to separate themselves from the action of doing mathematics (learning mathematical formulas and equations) and reflect on the results and the processes. This does not suggest it is not important to be able to solve a problem in Continuous Time Signals and Systems courses, but recommends that being able to reflect on the action of solving a problem is equally important for students. I argue that this ability must be the enduring understanding for all the topics related to Continuous Time Signals and Systems courses. Once this ability is developed, I contend that students can solve, explain, and apply any problem related to these courses whenever they have the related mathematical formulas and sometime even without.

Nasr (2007) conducted a study on understanding faulty reasonings of undergraduate aeronautical engineering students in Signals and Systems courses. His study was mainly focused on students' concepts related to system analysis. This study is focused on understanding undergraduate electrical engineering students' problematic reasonings in Signals and Systems courses. This study was focused on both signals and systems analysis and most of the findings are related to signal analysis. Nasr (2007) reported that students used faulty reasonings in problems related to concepts of linearity, time-invariance, convolution (specifically with the introduction of new symbol, τ), Laplace transform (specifically with the region of convergence), and Bounded-input-bounded-output stability of a system. This study also shows that students exhibit the use of problematic reasonings in problems related to convolution and time-invariance of the system. Participants of this study did not show problematic reasonings in finding linearity of the system. In this study, students' concepts related to Laplace transform and Bounded-Input-Bounded-Output stability of a system were not checked. Nasr (2007) suggests students use the reasoning resources based on the features of the given structure of problem or input-output pairs and are able to solve the problems related to system analysis correctly sometimes even with the lack of correct knowledge. Some of the frequently invoked resources are interval matching readout strategy and symmetry (Nasr, 2007). A student using a problematic reasoning and still correctly answering the given question was not observed in this study. The structure of questions being asked for this study can be a reason for the absence of this observation. This study suggests that students showed problems in solving questions related to convolution because they mixed up the operation of convolution with the operation of multiplication and showed problems in solving questions related to time-invariance because they mixed up the concept of time-invariance with the literal meaning of time-invariance.

Lesh, Post, and Behr (1987) interviewed fourth to eighth graders to investigate their understanding of applied mathematics, proportional reasoning, and rational numbers. They observed that the participants exhibited lack of understanding of the representations and translations of mathematical concepts. The results of this study show that students struggle with translation of signals from one representation to the other within a domain and between domains. Although this study is focused on undergraduate electrical engineering students' (not fourth to eighth graders') understanding of Signals and Systems (not mathematics) related topics, the results are consistent with the Lesh, Post, and Behr's (1987) findings. This suggests that may be students struggle in Signals and Systems courses because the skills required to understand the course content are not well acquired in the earlier classes. Learning the skills to translate and represent a mathematical concept in all five forms (graph, equation, symbol, real world, manipulative) suggested by Lesh, Post, and Behr (1987) will be a good preparation to excel in courses like Signals and Systems.

Dreyfus (1991) suggests that reflection on one's mathematical experiences is important in learning advanced mathematical processes. He proposes that an ability to check the solution through an alternative method is important for meaningful learning and students should consciously practice to understand representations and abstractions of advanced mathematical concepts. According to Dreyfus (1991), students can only learn advanced mathematical concepts through the conscious interaction of a large number of mathematical and psychological processes in their minds. These processes include representation, translation, modeling, generalizing, and synthesizing. Dreyfus's (1991) study is focused on high school and college students' understanding of Calculus related topics. The main findings of this study are not that the participants could not solve a particular problem due to their inability to perform calculus related operations but lack of abilities to translate between different representations of the same signal led to incorrect solutions. This suggest that although students might have learned to perform calculus related operations like integration, lack of ability to translate the given function (signals) hold students from solving Signals and Systems related problems. Developing an ability to reflect on the mathematical processes and verifying the answer from more than one method might help students to excel in courses like Signals and Systems. In Signals and Systems courses, students must attempt to challenge themselves to do multiple translations of the same concept and make connections between them with mathematical reasoning.

5.6 Future Work

The promotion of the 'conceptual change kind of learning' involves discussion of several critical issues including (i) In what ways is knowledge misunderstood, (ii) Why misunderstood knowledge is difficult to change, (iii) What comprises a change in prior knowledge, and (iv) how can the instruction be designed to promote conceptual change (Chi, 2008). This study is a step towards understanding the ways in which the knowledge is misunderstood. There are yet many areas to be explored before we can completely ensure conceptual learning of Continuous Time Signals and Systems course content. In this section, I will discuss some research opportunities that are created as a result of the knowledge unraveled in this study. Future research on learning Continuous Time Signals and Systems courses may include:

1. For this study, the analysis of the collected data was focused on the problematic reasonings of students, and what an incorrect response looks like in Continuous Time Signals and Systems courses. A future study (from the same collected data) that will complement this study and will help to better understand students' understanding of this course will be to look at what unproblematic reasonings of students look like when they engage with the same content. The data will be analyzed in the similar way it was done for this study except that this time except that the focus will be shifted from incorrect to correct responses and centered on what reasonings students use when they correctly solve the given problem. The results from this study will bring forward students' successful approaches to engage with Continuous Time Signals and Systems course content and will help in designing effective pedagogy for the students. A comparison of problematic and unproblematic reasonings might suggest a way to efficient fix-up of the problematic reasonings.
2. The data analysis for this study mainly focused on the identification of the problematic reasonings employed by students within Continuous Time Signals and Systems courses. An important area for future work from similar kind of data collected is to capture the problem solving strategies of students in Continuous Time Signals and Systems courses. These include studies like,
    a. Investigate how the students talk through the process (for example, linear, negotiated, etc) of problem solving within Continuous Time Signals and Systems courses.

b. Explore how the problem-solving approaches of the students who correctly solve a problem differ from the approaches of the students who incorrectly solve the same problem.

c. Explore the resources (for example, Fourier Transform table, calculator, computer, etc.) that the students rely on to solve a particular problem in Continuous Time Signals and Systems courses, how they make use of these resources, and how the use of these resources influence the problem solving strategies of the students.

3. An important area for the research in conceptual understanding of any concept is to find what makes that concept hard to understand (Perkins, 2007). Perkins (2007) suggests that the answer can be achieved by the identification of distinct patterns of how a student constructs meanings and concepts. Montfort (2011) puts forward a similar approach of answering this question. He recommends finding same student's correct and incorrect responses (understanding), identifying the connection between the incorrect and correct responses, and exploiting the connection to answer why a particular topic is difficult to understand for that particular student. The knowledge of what a student understands better empowers an educator to better teach any concept with the knowledge of i) potential areas to build analogies on, ii) to create references for learning, and iii) recognize how a student switches from one version of understanding to another in the course of problem solving, communicating, or learning.

4. Future research in continuation of this study will be establishing generalizability of the findings of this study (Montfort, Brown, & Pollock, 2009). Generalizability of the

findings of this study will help to address some methodological concerns that arise from the findings of any similar qualitative research on conceptual understanding. These include participant's ability to describe his/her reasonings accurately, influence of the communication skills of a participant to display his/her conceptual understanding, influence of the presence of the researcher on the participants' responses. A few ways to achieve this are to

a) Investigate the differences in the use of the problematic reasonings among undergraduate electrical engineering students in different places (universities, countries, etc.)
b) Investigate the difference in the use of problematic reasonings among students with different grades in Continuous Time Signals and Systems courses (Montfort, Brown, & Pollock, 2009)
c) Explore factors that play a role in the inculcation of the use of particular reasonings among different students in different places (Montfort, Brown, & Pollock, 2009), and
d) Develop a test (or revise SSCI) for quantitative analysis of conceptual understanding of undergraduate electrical engineering students within Continuous Time Signals and Systems course content

5. Another important area for further research in conceptual learning of Continuous Time Signals and Systems courses is a longitudinal study to explore how conceptual knowledge of undergraduate electrical engineering students of topics covered in Continuous Time Signals and Systems courses evolves as they move forward in electrical engineering program.

6. For students who have taken one year of calculus and analytic geometry in high school, Iris University provides an intensive five-week fast track calculus course in which students review differential and integral calculus, cover all of multivariable calculus, and become familiar with the computer implementation of mathematics. Upon successful completion of Fast-Track Calculus, the student receives 15 hours of academic credit, and is able to enter Sophomore-level mathematics courses as a freshman. In the future, it will be interesting to explore the difference in the understanding of Continuous Time Signals and Systems course content between students with a non-traditional mathematical training and those without.

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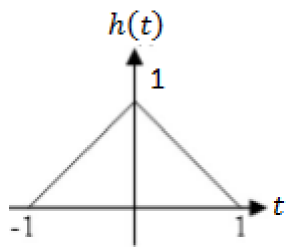
APPENDICES

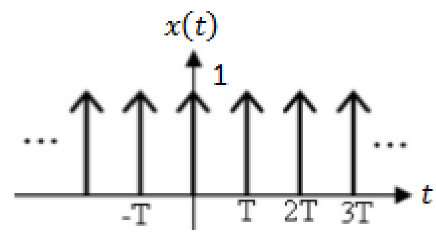
Appendix A - Protocol-A (First Pilot Study)

- The point of the interview is for me to try to understand your thought processes as you work through a question.
- Please try to explain your reasoning out loud.
- Don't worry about whether you are right or wrong.
- You are welcome to use whatever tools you need to work these out (paper, calculator, etc)
- I may ask you follow up questions as you answer the question.

Question 1

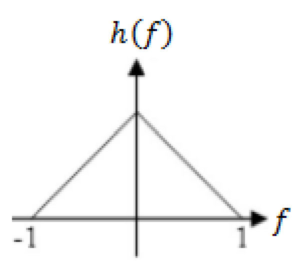
(a) For h(t) and x(t) given below,

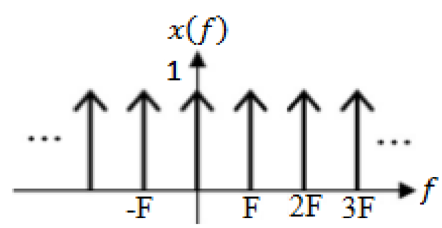




plot $y(t)$ such that
$y(t) = h(t) * x(t)$ and T=2.

(b) If x(t) and h(t) are the same functions but in frequency domain instead of time domain, as shown below, how will the shape of y(f) change if y(f) is still convolution of x(f) and h(f). Plot y(f).





Question 2

Draw magnitude plot and phase plot of exponential Fourier series of the given voltage signal, *v(t)*

$$v(t) = \cos\left(t + \frac{\pi}{4}\right) + \ 3\sin\,(7t)$$

Plot magnitude and phase of Fourier transform of *v(t)*.

If units of *v*(*t*) is volts, what will be the units of Fourier series and Fourier transform of *v*(*t*).

What similarities, differences, or relation exist between Fourier series and Fourier transform of *v*(*t*)?

Question 3

[1] Plot the magnitude and phase of
   i. Fourier series of $x_1(t)$
   ii. Fourier transform of $x_1(t)$

$$x_1(t) = Re(e^{j\left(5t+\frac{\pi}{3}\right)}) + Im(e^{j\left(4t-\frac{\pi}{4}\right)})$$

[2] Plot the magnitude and phase of
   i. Fourier series of $x_2(t)$
   ii. Fourier transform of $x_2(t)$

$$x_2(t) = \cos(\pi t) + e^{j2\pi t}$$

[3] Plot the magnitude and phase of
   i. Fourier series of $x_3(t)$
   ii. Fourier transform of $x_3(t)$

$$x_3(t) = \sin\,(\frac{\pi}{6}t - \frac{\pi}{6}) + e^{j\frac{\pi}{3}}$$

[4] Plot the magnitude and phase of
   i. Fourier series of $x_4(t)$
   ii. Fourier transform of $x_4(t)$

$$x_4(t) = e^{j\frac{\pi}{2}}[\delta(t+\pi) - \delta(t-\pi)]$$

Question 4

a) For $r(t) = tu(t)$, plot

i. $r(t)$
ii. $x(t) = r(t)u(2-t)$
iii. $d(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$
iv. $g(t) = d(t)u(t-1)$
v. $z(t) = g(-2t+3)$

b) For $x(t) = 5sin\omega t$

i. Plot $x(t)$ with respect to ωt. Completely label the graph.
ii. Integrate $x(t)$ with respect to t, such that

$$y(t) = \int 5sin\omega t\, dt$$

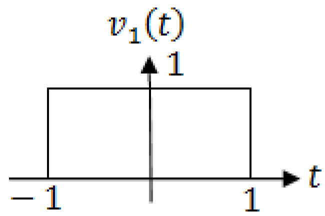
iii. Plot $y(t)$ with respect to ωt
iv. Plot $y(t)$ with respect to ω for $t = 1$

Question 5

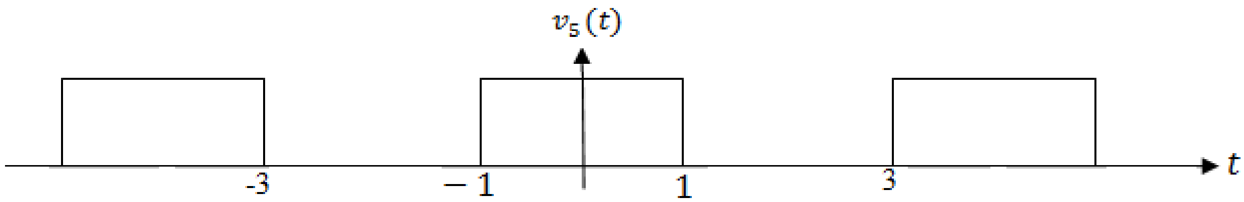
a) What will be the Fourier transform of $g(t) = t^2$
b) What will be the Fourier transform of $x(t) = t^2u(t)u(1-t)$? Show some initial steps to find Fourier transform of $x(t)$ by integration.
c) What will be the Fourier transform of $z(t) = t^2\delta(t-1)$

Question 6

a) Let $s_1(t)$ be the integral of $v_1(t)$ shown below, find and plot $s_1(t)$.

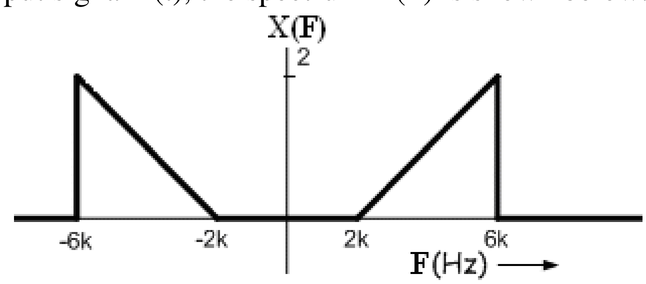


b) Let $s_5(t)$ be the integral of the periodic signal $v_5(t)$ shown below, find and plot $s_5(t)$.

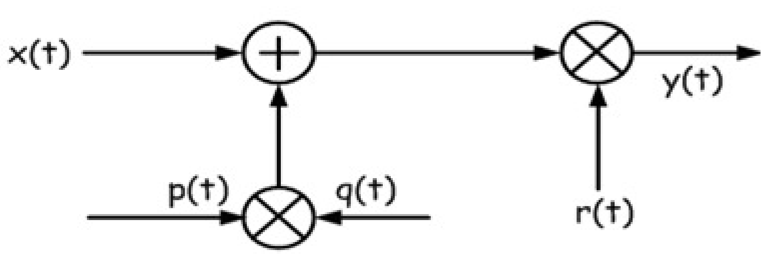


Question 7

For a real-valued input signal x(t), the spectrum X(F) is shown below.



This signal x(t) is combined with different cosine signals using one adder and two multipliers as shown in the block diagram below.



where, p(t) = cos(6000πt), q(t) = cos(8000πt) and r(t) = cos(12000πt)

Sketch and completely label the spectrum of y(t)

Protocol-B (First Pilot Study)

- The point of the interview is for me to try to understand your thought processes as you work through a question.
- Please try to explain your reasoning out loud.
- Don't worry about whether you are right or wrong.
- You are welcome to use whatever tools you need to work these out (paper, calculator, etc)
- I may ask you follow up questions as you answer the question.

Question 1

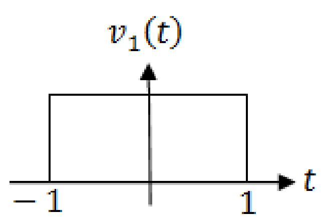
Following are some voltage signals.

i. $v_2(t) = \delta(t)$

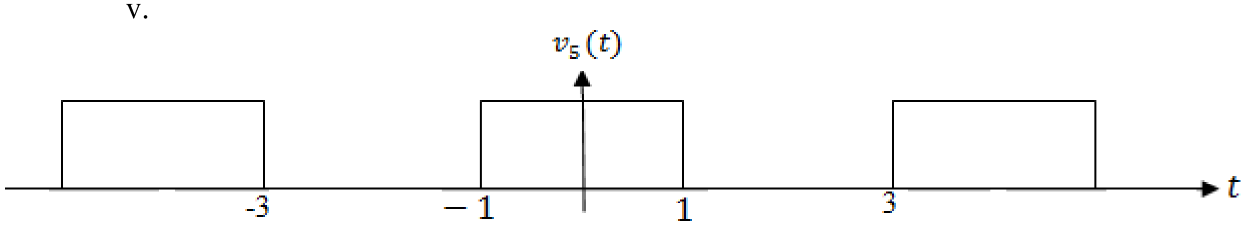
ii. $v_3(t) = 1$

iii. $v_4(t) = 0.504 \sum_{k=-\infty}^{\infty} \frac{1}{1+j4k} e^{jkt}$

iv.



v.



a) Find Fourier transforms of these signals

b) Explain which frequencies are present in these signals

c) Explain which frequencies are not present in these signals

Question 2

a) $x_1(t)$ is a continuous time signal shown below:

$$x_1(t) = \cos(50\pi t) + \cos(170\pi t) + \cos(290\pi t)$$

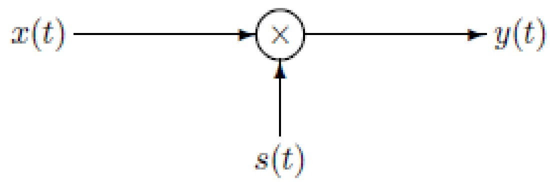
i. If $x_1(t)$ is sampled with sampling frequency, F_S=60 cycles/sec, plot Fourier transform of the sampled signal.

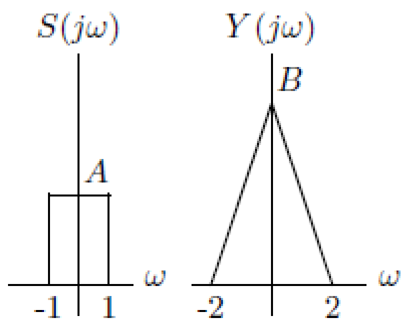
ii. If $x_1(t)$ is sampled with sampling frequency, F_S=25 cycles/sec, plot Fourier transform of the sampled signal.

b) Two discs are rotating with different speeds in the same direction in a dark area. One is moving with 3780 rpm and other with 6300 rpm. Two numbers are written on the rotating discs that can be read by putting flashlight on these discs. The flashlight can be tuned at any single frequency with integer value between 15 flashes per second to 25 flashes per second. We intend to tune this flashlight at a frequency with which both numbers can be read. Which frequency for flashlight is most suitable to read the numbers correctly?

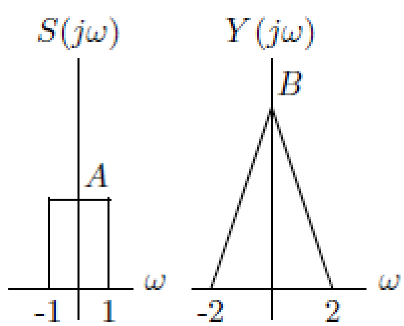
Question 3

a) Let y(t) be the output of a multiplier as shown below. Fourier transforms of s(t) and y(t) are shown below as S(jω) and Y(jω) respectively. A and B are unknown constants. Plot the Fourier transform of x(t).



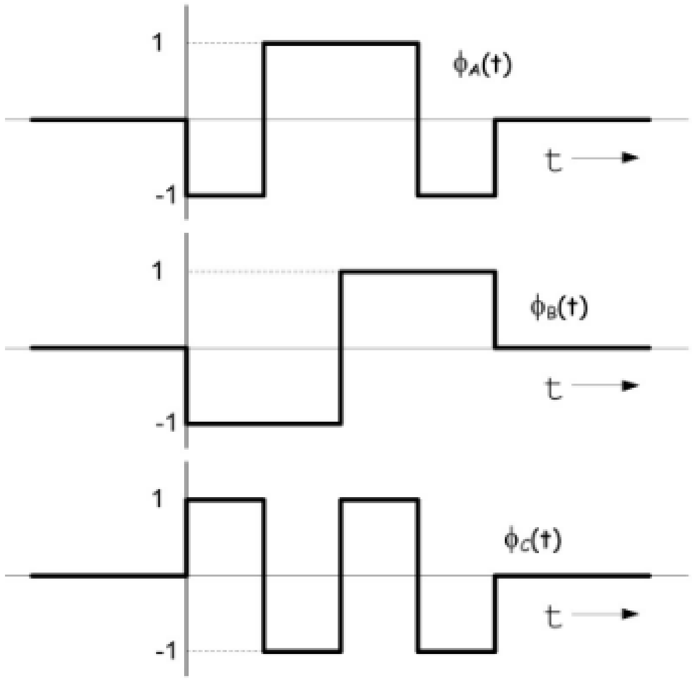


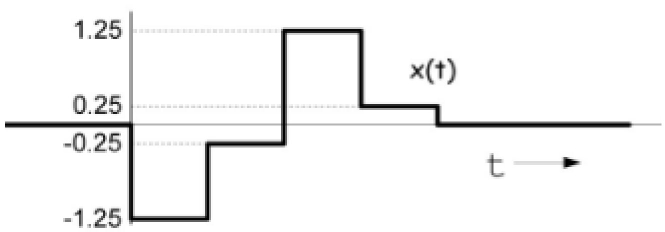
b) If s(t) is the impulse response of an LTI system and input to the system is x(t), such that S(jω) and Y(jω) are shown below. Plot the Fourier transform of x(t)



Question 4

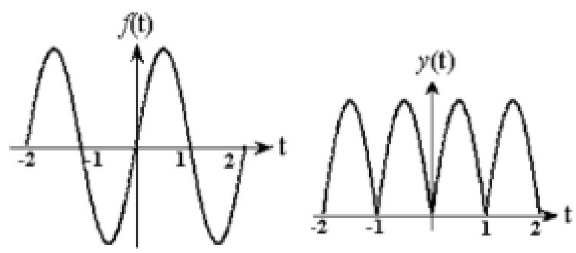
Three signals $\emptyset_A(t), \emptyset_B(t), \emptyset_C(t)$ are shown below. A new signal $x(t)$ is formed by adding amplitude scaled versions of two of these three signals. How will you determine which two signals are combined to get $x(t)$.





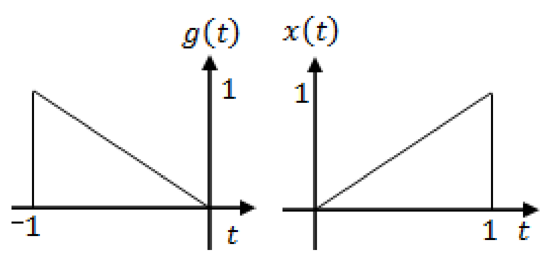
Question 5

a) Explain why do most of the systems in signal processing are LTI systems.
b) The impulse response of a system is $h(t) = 5\,\mathrm{sgn}(t-2)$, explain whether this system is
   i. linear
   ii. time-invariant
   iii. LTI
c) The system in the diagram shown below is used to convert an ac voltage signal to dc voltage signal. The input *f*(t) and output *y*(t) of the system are shown below. Explain whether this system
   i. linear
   ii. time-invariant
   iii. LTI



Question 6

Two signals x(t) and g(t) are shown below such that $y(t) = x(t) * g(t)$, find area under $y(t)$ without solving convolution integral.



What will be Y(0) (Fourier transform of $y(t)$ evaluated at 0 frequency)

Appendix B - Protocol A (Second Pilot Study)

My name is Farrah Fayyaz and I am a doctoral candidate in the School of Engineering Education at Purdue University. Thank you for agreeing to participate in our study on identification of persistently difficult concepts in learning Signals and Systems courses among undergraduate electrical engineering students. We will review the study procedures and what your participation entails before you begin the study. First, I need you to complete a consent form. I would like to explain the purpose and procedures of the study and give you an opportunity to ask questions. Dr. [redacted] is a key personnel in this project. However, neither the key personnel nor any other faculty at your school will have any information about your participation or have access to any identifiable data collected through this study.

Farrah Fayyaz reviews each section of the consent form.

Are there any questions?

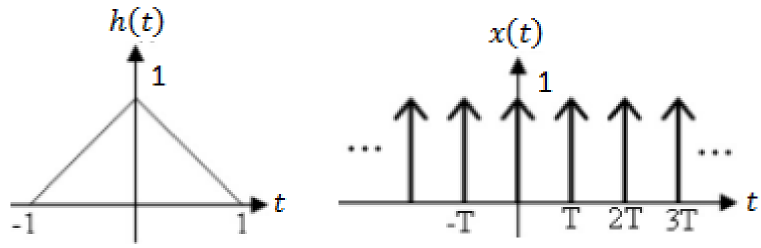
Please sign and date the consent form if you are still willing to participate in this study.

Let me explain the full procedure to you. The point of the interview is for me to try to understand your thought processes as you work through a question. Please try to explain your reasoning out loud. Do not worry about whether you are right or wrong. You are welcome to use calculator, if needed. I may ask you follow up questions as you answer the question. You will write your responses with pen on this tablet (Microsoft Surface Pro). If you are not comfortable with using it, we can spend first five minutes in familiarizing with it. I will give you a $20 voucher at the end of the interview that you can redeem with the business office secretary. The $20 compensation is for the complete interview. If the interview takes longer than an hour and you are still willing to give extra time and complete the interview, you will not get extra money.

Please put the pen on the tablet while referring to anything. If you are not doing so, I will keep reminding you. Don't get nervous about that.

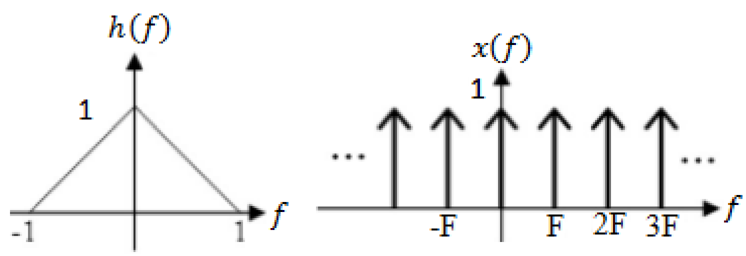
Question 1

For h(t) and x(t) given below,



a) Plot and completely label $y(t)$ such that $y(t) = h(t) * x(t)$ and T=1.
b) Make a plot that shows how the shape of y(t) will change with increase in the value of T
c) Make a plot that shows how the shape of y(t) will change with decrease in the value of T
d) Explain the relation between the transforms of h(t) and y(t).
e) Explain the relation between the lengths of transforms of h(t) and y(t).

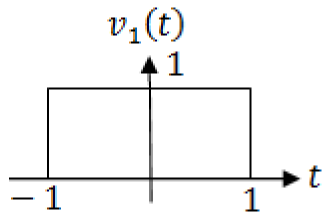
If x(t) and h(t) are the same functions but in frequency domain instead of time domain, as shown below,



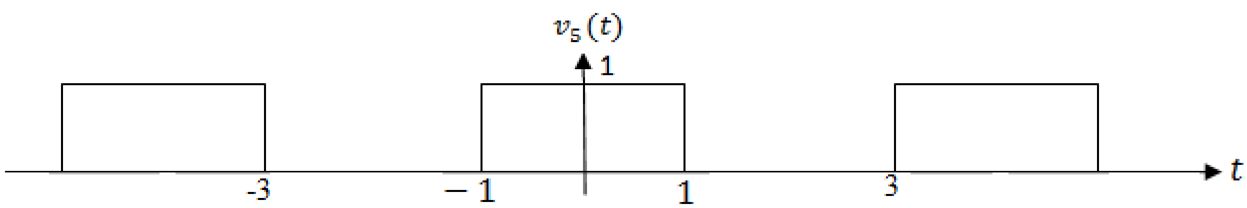
a) Plot y(f) which is convolution of x(f) and h(f), for F=2.
b) Make a plot that shows how the shape of y(f) will change with increase in F.
c) Explain the relation between the inverse transforms of h(f) and y(f) when F is greater than 2
d) Make a plot that shows how the shape of y(f) will change with decrease in F.
e) Explain the relation between the inverse transforms of h(f) and y(f) when F is less than 2

<u>Question 2</u>

a) Let $s_1(t)$ be the integral of $v_1(t)$ shown below, find and plot $s_1(t)$.



b) Let $s_5(t)$ be the integral of the periodic signal $v_5(t)$ shown below, find and plot $s_5(t)$.



Question 3

Following the given voltage signals,

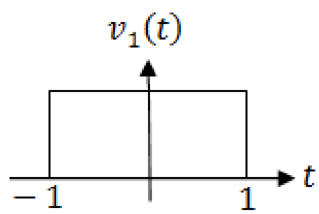
a) Plot Fourier transforms of these signals. Completely label the values and units on x- and y-axes.
b) Explain which frequencies are present in these signals
c) Explain which frequencies are not present in these signals

i. $v_2(t) = \delta(t)$

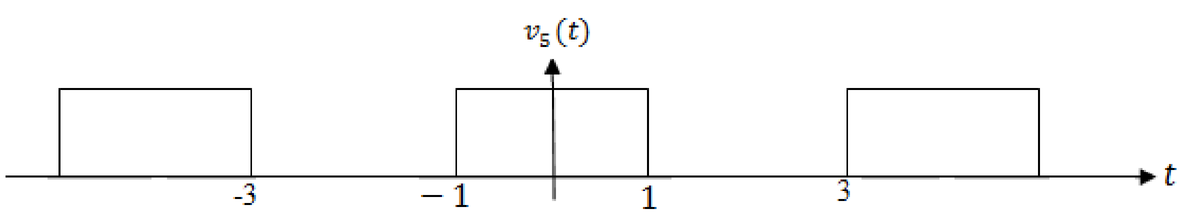
ii. $v_3(t) = 1$

iii. $v_4(t) = 0.504 \sum_{k=-\infty}^{\infty} \frac{1}{1+j4k} e^{jkt}$

iv.



v.

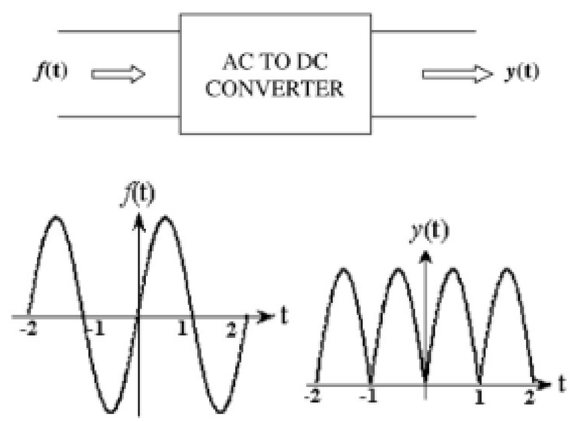


<u>Question 4</u>

a) Explain why do most of the systems in signal processing are LTI systems.

b) The impulse response of a system is $h(t) = 5\,\mathrm{sgn}(t-2)$. Explain whether this system is
   i. Linear
   ii. time-invariant
   iii. LTI

c) The system in the diagram shown below is used to convert an ac voltage signal to dc voltage signal. The input *f*(t) and output *y*(t) of the system are shown below. Explain whether this system is
   i. Linear
   ii. time-invariant
   iii. LTI



Protocol B (Second Pilot Study)

Question 1

a) For $v(t) = \cos\left(t + \frac{\pi}{4}\right) + 3\sin(7t)$,

i. **Plot the magnitude and phase** of Fourier series of $v(t)$

ii. **Plot the magnitude and phase** of Fourier transform of $v(t)$

iii. If units of $v(t)$ is volts, what will be the units of Fourier series and Fourier transform of $v(t)$.

iv. Explain what similarities, differences, and relation exist between Fourier series and Fourier transform of $v(t)$?

b) For $x_1(t) = Re(e^{j\left(5t+\frac{\pi}{3}\right)}) + Im(e^{j\left(4t-\frac{\pi}{4}\right)})$, **plot the magnitude and phase of** Fourier series of $x_1(t)$

c) For $x_2(t) = \cos(\pi t) + e^{j2\pi t}$, **plot the magnitude and phase of** Fourier transform of $x_2(t)$

d) For $x_3(t) = \sin\left(\frac{\pi}{6}t - \frac{\pi}{6}\right) + e^{j\frac{\pi}{3}}$, **plot the magnitude and phase of** Fourier series of $x_3(t)$

e) For $x_4(t) = e^{j\frac{\pi}{2}}[\delta(t + \pi) - \delta(t - \pi)]$, **plot the magnitude and phase of**

i. Fourier series of $x_4(t)$

ii. Fourier transform of $x_4(t)$

Question 2

a) For $r(t) = tu(t)$, plot (also label the axes)

i. $r(t)$

ii. $x(t) = r(t)u(2-t)$

iii. $d(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$

iv. $g(t) = d(t)u(t-1)$

v. $z(t) = g(-2t+3)$

b) For $x(t) = 5sin\omega t$

i. Plot $x(t)$ with respect to ωt. Label the graph and axes.

ii. Integrate $x(t)$ with respect to t, such that

$$y(t) = \int 5sin\omega t \, dt$$

iii. Plot $y(t)$ with respect to ωt. Label the graph and axes

iv. Plot $y(t)$ with respect to ω for $t = 1$. Label the graph and axes

Question 3

a) Plot (also label the axes) $g(t) = t^2$

b) What will be the Fourier transform of $g(t)$

c) What will be the Fourier transform of $x(t) = t^2 u(t)u(1-t)$? Show some initial steps to find Fourier transform of $x(t)$ by integration.

d) Find Fourier transform of $z(t) = t^2 \delta(t-1)$

e) Plot (also label the axes) Fourier transform (magnitude and phase) of $z(t)$

Question 4

a) Explain what you understand about convolution.

b) How would you explain convolution to a non-engineer

c) Can you suggest some applications of convolution.

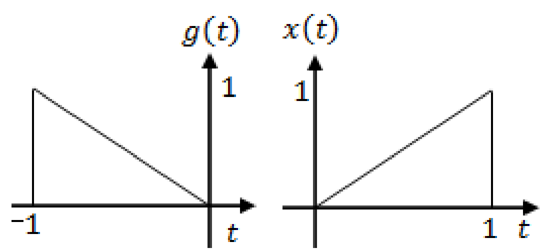
d) Can you suggest some non-engineering applications of convolution.

e) Explain what you understand about Fourier analysis.

f) Can you suggest some applications of Fourier analysis.

g) Can you suggest some non-engineering applications of Fourier analysis.

h) Two signals x(t) and g(t) are shown below such that $y(t) = x(t) * g(t)$, find area under $y(t)$ without solving convolution integral.

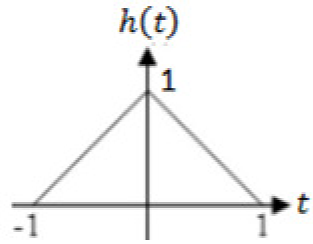


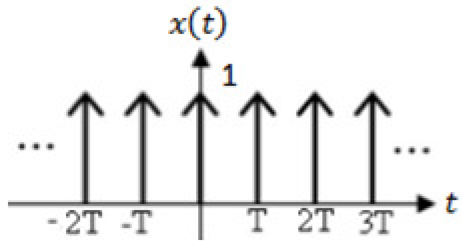
i) What will be Y(0) (Fourier transform of $y(t)$ evaluated at 0 frequency)

Appendix C - Protocol A (Actual study)

Question 1

For two signals, h(t) and x(t) given below,



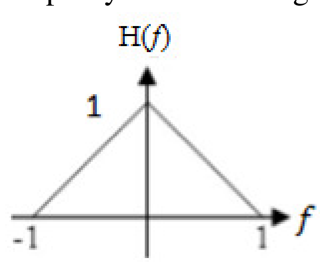


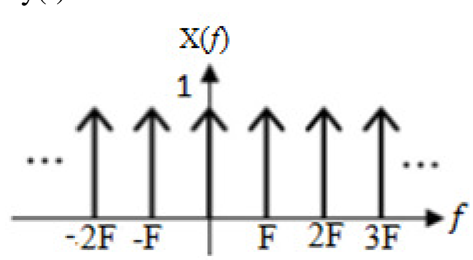
f) Explain to the researcher how you will plot $y(t)$ such that $y(t) = h(t) * x(t)$ and T=2.

g) Forget that signal y(t) is obtained from the convolution of x(t) and h(t), use h(t) given above and y(t) from part (a),
   a. Explain to the researcher how you know whether there is any relation between h(*t*) and y(*t*).
   b. Explain to the researcher how you know whether the change in the shape of y(t) with the change in the value of T (greater than 2 or less than 2) changes the relation between h(*t*) and y(*t*) .

h) Frequency and time are inversely proportional to each other.
   a. Explain to the researcher whether you see this inverse relation between the length of h(t) and the bandwidth of Fourier transform of h(t). *(Note: length of h(t) is 2, i.e. from -1 to 1)*
   b. Explain to the researcher whether you see this inverse relation between the length of y(t) and the bandwidth of Fourier transform of y(t).

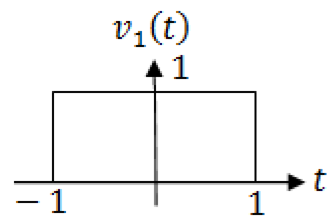
i) Consider that signals *x* and *h* are same as given above but in frequency domain instead of time domain as shown below. And $y'(f) = h(f) * x(f)$. Explain to the researcher in detail how you know whether the plot of y'(*f*) (for F =2) is different from the plot you made of signal y(*t*) above.



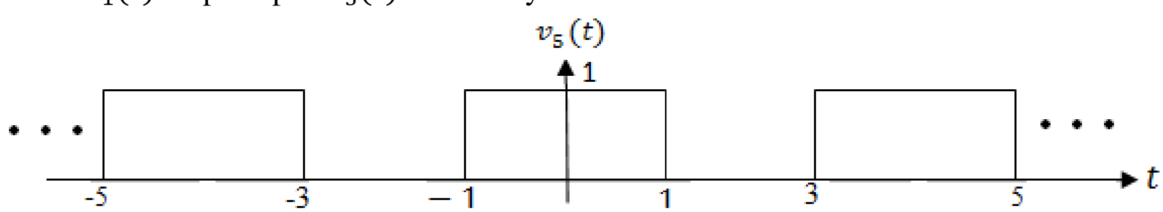


Question 2

a) Let $s_1(t)$ be the integral of the signal $v_1(t)$ shown below, explain to the researcher in detail how you will find and plot $s_1(t)$.



b) The signal $v_5(t)$ given below is obtained by making the above signal $v_1(t)$ periodic. Let $s_5(t)$ be the integral of $v_5(t)$. Explain to the researcher in detail whether the plot of $s_1(t)$ helps to plot $s_5(t)$. How do you know?



c) Explain superposition principal to the researcher in detail. Use examples of $v_1(t)$ and $v_5(t)$ or $s_1(t)$ and $s_5(t)$ to illustrate your explanation.

<u>Question 3</u>

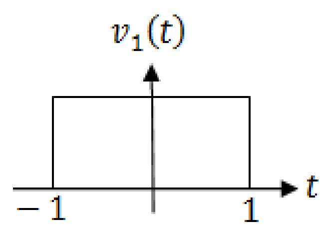
a) For each of the voltage signals given below, explain to the researcher in detail how you know which frequencies are present and which are not present in each signal.

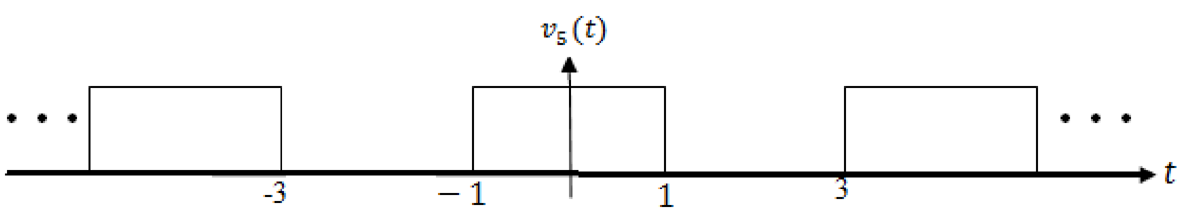
i. $v_2(t) = \delta(t)$

ii. $v_3(t) = 1$

iii. $v_4(t) = 0.504 \sum_{k=-\infty}^{\infty} \frac{1}{1+j4k} e^{jkt}$

b) Explain to the researcher in detail whether the knowledge of frequencies present in an aperiodic signal helps to determine the frequencies present in another signal which is obtained by making that aperiodic signal periodic. An example of such signals is given below where periodic signal $v_5(t)$ is obtained by making aperiodic signal $v_1(t)$ periodic.





Question 4

1. Explain to the researcher why most of the systems in signal processing are LTI systems.

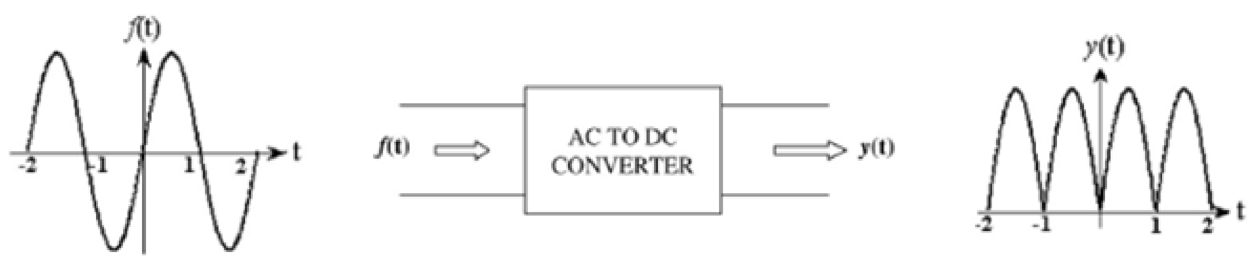
2. The impulse response of a system is $h(t) = 5\,\mathrm{sgn}(t-2)$. Explain to the researcher in detail how you know whether this system is:
   i) Linear

   ii) time-invariant

3. An ac-to-dc converter converts an ac voltage signal to a dc voltage signal as shown in the diagram below. Explain to the researcher in detail how you know whether this system is:

   i) Linear

   ii) time-invariant



Protocol B (Actual study)

Question 1

[1] For $x_4(t) = e^{-j\frac{\pi}{2}}[\delta(t+\pi) - \delta(t-\pi)]$

a. Explain to the researcher in detail how you will plot the magnitude and phase of Fourier transform of $x_4(t)$.

b. Explain to the researcher in detail whether the plot of Fourier transform of a signal provides information about how you will plot the Fourier series of the same signal. Use signal $x_4(t)$ given above as an example to explain your reasoning to the researcher.

[2] For $x_3(t) = \sin\left(\frac{\pi}{6}t - \frac{\pi}{6}\right) + e^{j\frac{\pi}{3}}$, explain to the researcher in detail how you will plot the magnitude and phase of Fourier transform of $x_3(t)$.

[3] For $v(t) = \cos\left(t + \frac{\pi}{4}\right) + 3\sin(7t)$

i. Explain to the researcher in detail how you will plot the magnitude and phase of Fourier series of $v(t)$.

ii. Explain to the researcher in detail whether the plot of Fourier series of a signal provides information about how you will plot the Fourier transform of the same signal. Use signal $v(t)$ given above as an example to explain your reasoning to the researcher.

iii. If units of $v(t)$ is volts, explain to the researcher what will be the
   a. Units of Fourier series of $v(t)$. How do you know?
   b. Units of Fourier transform of $v(t)$. How do you know?

Question 2

a) For $r(t) = tu(t)$ and $g(t) = [r(t) - r(t-1)]u(-t+2)$, describe to the researcher in detail how you will plot $g(-2t+2)$.

b) Explain to the researcher in detail how you will plot $h(t)$ where $h(t) = \sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$. As you plot, also describe as much as possible about how you will label x-axis.

c) Explain the concept of 'time shift' to the researcher. Use the example of the plot of $h(t)$ you made above to explain this concept.

d) Explain the concept of 'phase shift' to the researcher. Use the example of the plot of $h(t)$ you made above to explain this concept.

e) Describe to the researcher in detail how you know whether there is any relation between the concept of phase shift and the concept of phase difference.

Question 3

a) Explain to the researcher how you will plot signal $d(t)$ such that $d(t) = t^2$.

b) Explain to the researcher how you go about finding Fourier transform of any given signal. Use the example of signal $d(t)$ to explain your answer.

c) Explain to the researcher whether you will modify your procedure to find Fourier transform of a signal described above if instead of signal $d(t)$ you have signal $x(t)$ such that $x(t) = t^2u(t)u(1-t)$.

d) Explain to the researcher how you will plot Fourier transform of signal $z(t)$ such that $z(t) = t^2\delta(t-1)$.

Question 4

a) Explain the concept of convolution to the researcher assuming the researcher is a first-year engineering student.
b) Assume the researcher is a non-engineer. Explain how you will help the researcher to identify problems or situations where a non-engineer can apply the concept of convolution.
c) Explain the concept of Fourier analysis to the researcher in detail assuming the researcher is a first-year engineering student.
d) Describe whether you will modify your explanation of the concept of Fourier analysis for best understanding of the researcher if the researcher is not an engineer.
e) For any given signal $y(t)$, explain to the researcher how you know whether there is any relation between area under the signal, i.e., $\int_{-\infty}^{\infty} y(t)dt$ and Fourier transform of the signal evaluated at zero frequency, i.e., Y(0).

Appendix D - IRB approval letter for the pilot study

PURDUE
UNIVERSITY

HUMAN RESEARCH PROTECTION PROGRAM
INSTITUTIONAL REVIEW BOARDS

To:	RUTH STREVELER ARMS
From:	JEANNIE DICLEMENTI, Chair Social Science IRB
Date:	12/17/2013
Committee Action:	**Approval**
IRB Action Date	12/17/2013
IRB Protocol #	1312014282
Study Title	Examination of problems in learning Signals and Systems courses using think-aloud protocols
Expiration Date	12/16/2014

Following review by the Institutional Review Board (IRB), the above-referenced protocol has been approved. This approval permits you to recruit subjects up to the number indicated on the application form and to conduct the research as it is approved. The IRB-stamped and dated consent, assent, and/or information form(s) approved for this protocol are enclosed. Please make copies from these document(s) both for subjects to sign should they choose to enroll in your study and for subjects to keep for their records. Information forms should not be signed. Researchers should keep all consent/assent forms for a period no less than three (3) years following closure of the protocol.

Revisions/Amendments: If you wish to change any aspect of this study, please submit the requested changes to the IRB using the appropriate form. IRB approval must be obtained before implementing any changes unless the change is to remove an immediate hazard to subjects in which case the IRB should be immediately informed following the change.

Continuing Review: It is the Principal Investigator's responsibility to obtain continuing review and approval for this protocol prior to the expiration date noted above. Please allow sufficient time for continued review and approval. No research activity of any sort may continue beyond the expiration date. Failure to receive approval for continuation before the expiration date will result in the approval's expiration on the expiration date. Data collected following the expiration date is unapproved research and cannot be used for research purposes including reporting or publishing as research data.

Unanticipated Problems/Adverse Events: Researchers must report unanticipated problems and/or adverse events to the IRB. If the problem/adverse event is serious, or is expected but occurs with unexpected severity or frequency, or the problem/even is unanticipated, it must be reported to the IRB within 48 hours of learning of the event and a written report submitted within five (5) business days. All other problems/events should be reported at the time of Continuing Review.

We wish you good luck with your work. Please retain copy of this letter for your records.

Ernest C. Young Hall, 10th Floor - 155 S. Grant St. - West Lafayette, IN 47907-2114 - (765) 494-5942 - Fax: (765) 494-9911

Appendix E - IRB approved consent form for the pilot study

For IRB Use Only

APPROVED
DEC 17 2013
EXPIRES 12-16-14
PURDUE UNIVERSITY
INSTITUTIONAL REVIEW BOARD

RESEARCH PARTICIPANT CONSENT FORM

Examination of problems in learning Signals and Systems courses using think-aloud protocols
Principal Investigator: Dr Ruth Streveler
School of Engineering Education
Purdue University

What is the purpose of this study?

The purpose of this research is to examine Electrical Engineering students' thought processes while solving a given problem in Signals and Systems (ECE 301) courses. Understanding these thinking processes will allow us to better teach future electrical and computer engineering undergraduate students.

What will I do if I choose to be in this study?

You participated in a 90-minute long one-on-one interview. The researcher asked you questions related to the topics covered in Signals and Systems (ECE [redacted]) courses. You were asked to think-out loud while responding to the questions. Additionally, you were asked to write your working on the worksheets. The interview session was audio recorded.

How long will I be in the study?

Your participation in this study lasted for approximately ninety minutes in length.

What are the possible risks or discomforts?

All research carries risk. The standard for minimal risk is that which is found in everyday life. With the research team's efforts to maintain confidentiality, risk of your identification is unlikely; however, there is a potential of breach of confidentiality with any research. Safeguards are in place to minimize the risk of breach of confidentiality, as outlined in the confidentiality section of this consent form. Risks greater than those encountered in everyday life are not anticipated. Your participation in the research study will have no effect upon your grades or class standing in any course.

Are there any potential benefits?

There are no direct benefits to participating in the study. However, having practice of describing your problem solving process could be useful in job interviews. Your responses may help to improve the development of content, assessment, and pedagogy that helps students better understand difficult concepts in Signals and Systems (ECE [redacted]) courses.

Will I receive payment or other incentive?

You were provided with snacks during the interview. There was no additional compensation for participation in this study. There were no extra costs to participants.

Will information about me and my participation be kept confidential?

Confidentiality regarding your participation will be maintained. The faculty at Purdue will not be given any knowledge of your participation in this study, and all reasonable efforts will be made to ensure that your data remains anonymous to them. Any notes associated with this interview will be used without reference to your name. Any information that can connect the data with you will be destroyed after the study is complete. The data collected in this interview may be used for future research purposes, but it will be de-identified and your identity will not be traceable from the data by anyone including any researcher in this study. Transcriptions will be made from the interview audio files, with all identifying information removed. Audio data will be stored on a password protected portable hard drive which will be kept in a locked filing cabinet in the PI's office at Purdue University. Your work sheets and consent forms will also be kept locked in the PI's office at Purdue University and retained indefinitely. Only Farrah Fayyaz will know the actual identity of the participants. Only the unidentifiable data will be available to the personnels within the research team of this study. No one outside this study will have access to even the de-identified data. The results of this study will be presented in a way that any information about the participants will be unidentifiable. The project's research records may be reviewed by department at Purdue responsible for regulatory and research oversight.

What are my rights if I take part in this study?

You do not have to participate in this research project. Your participation in this study was voluntary. You may choose not to participate or, if you had agreed to participate, you can withdraw your participation at any time without penalty or loss of benefits to which you are otherwise entitled. However, as described above, data will be de-identified such that the participants' names and other critical identification information will be removed, which means once the identity is removed from the data we will no longer be able to identify your data from any of the other data. This restricts your choice to withdraw from participating in the study only up to the time when the association of your name with the data is destroyed. Your participation will not affect any of your grades.

Who can I contact if I have questions about the study?

Farrah Fayyaz conducted the procedures of this study. For any questions related to that, she will be your first point of contact. You can contact her at ffayyaz@purdue.edu or [redacted] For any further questions, you can contact the principal investigator of this study, Dr. Ruth Streveler in School of Engineering Education at Purdue University at rastreve@purdue.edu or [redacted]

If you have questions about your rights while taking part in the study or have concerns about the treatment of research participants, please contact [redacted] Institutional Review Board (IRB) at [redacted] Research Protection Program at Purdue University at (765) 494-5942, email (irb@purdue.edu) or write to:

Human Research Protection Program - Purdue University
Ernest C. Young Hall, Room 1032
155 S. Grant St.,
West Lafayette, IN 47907-2114

Documentation of Informed Consent

I have had the opportunity to read this consent form and have the research study explained. I have had the opportunity to ask questions about the research study, and my questions have been answered. I am prepared to participate in the research study described above. I will be offered a copy of this consent form after I sign it.

________________________________ ____________________

Participant's Signature Date

____

Participant's Name

________________________________ ____________________

Researcher's Signature Date

Appendix F - IRB Approval Letter for the actual study

PURDUE
UNIVERSITY

HUMAN RESEARCH PROTECTION PROGRAM
INSTITUTIONAL REVIEW BOARDS

To:	RUTH STREVELER ARMS
From:	JEANNIE DICLEMENTI, Chair Social Science IRB
Date:	11/13/2013
Committee Action:	**Approval**
IRB Action Date	11/12/2013
IRB Protocol #	1310014094
Study Title	Identification of persistant misconceptions and difficult concepts in learning Signals and Systems courses among undergraduate electrical engineering students
Expiration Date	11/11/2014

Following review by the Institutional Review Board (IRB), the above-referenced protocol has been approved. This approval permits you to recruit subjects up to the number indicated on the application form and to conduct the research as it is approved. The IRB-stamped and dated consent, assent, and/or information form(s) approved for this protocol are enclosed. Please make copies from these document(s) both for subjects to sign should they choose to enroll in your study and for subjects to keep for their records. Information forms should not be signed. Researchers should keep all consent/assent forms for a period no less than three (3) years following closure of the protocol.

Revisions/Amendments: If you wish to change any aspect of this study, please submit the requested changes to the IRB using the appropriate form. IRB approval must be obtained before implementing any changes unless the change is to remove an immediate hazard to subjects in which case the IRB should be immediately informed following the change.

Continuing Review: It is the Principal Investigator's responsibility to obtain continuing review and approval for this protocol prior to the expiration date noted above. Please allow sufficient time for continued review and approval. No research activity of any sort may continue beyond the expiration date. Failure to receive approval for continuation before the expiration date will result in the approval's expiration on the expiration date. Data collected following the expiration date is unapproved research and cannot be used for research purposes including reporting or publishing as research data.

Unanticipated Problems/Adverse Events: Researchers must report unanticipated problems and/or adverse events to the IRB. If the problem/adverse event is serious, or is expected but occurs with unexpected severity or frequency, or the problem/even is unanticipated, it must be reported to the IRB within 48 hours of learning of the event and a written report submitted within five (5) business days. All other problems/events should be reported at the time of Continuing Review.

We wish you good luck with your work. Please retain copy of this letter for your records.

Ernest C. Young Hall, 10th Floor - 155 S. Grant St. - West Lafayette, IN 47907-2114 - (765) 494-5942 - Fax: (765) 494-9911

Appendix G - IRB approved Consent Form for the actual study

For IRB Use Only

APPROVED
NOV 12 2013
EXPIRES 11-11-14
PURDUE UNIVERSITY
INSTITUTIONAL REVIEW BOARD

RESEARCH PARTICIPANT CONSENT FORM

Identification of persistent misconceptions and difficult concepts in learning Signals and Systems courses among undergraduate electrical engineering students
Principal Investigator: Dr Ruth Streveler
School of Engineering Education
Purdue University

What is the purpose of this study?

The goal of this project is to understand the misconceptions and difficult concepts in learning Continuous-Time Signals Systems ([redacted]) course among undergraduate electrical and computer engineering students. Understanding these thinking processes will allow us to better teach future electrical and computer engineering undergraduate students.

What will I do if I choose to be in this study?

You will participate in a 1-hour long one-on-one interview. The researcher will ask you questions related to the topics covered in Continuous-Time Signals Systems [redacted]. You will be asked to think-out loud while responding to the questions. Additionally, you will be asked to write your working on a pdf file on Microsoft Surface Pro. You will be given some time in the beginning to get familiar with Microsoft Surface Pro, if you are unfamiliar with it. You will be encouraged to touch or circle any figure or equation you are referring to while responding. The interview session will be audio recorded. Additionally, screen of the Microsoft Surface Pro will be recorded. In order to better understand your preparation for the [redacted] content, we are asking for your permission to access your education records (grades in engineering, mathematics, and physics courses; Signals and Systems concept inventory scores; and Learning Style Index scores). Furthermore, we will ask you if you will want to participate (interview) in a follow-up study on how the conceptual understanding of the [redacted] course contents change as students progress in their academic career. You may still participate in the study even if you decline to let us access your education records or to participate in the follow-up interview. Selecting the options at the end of this consent form and signing this consent form is validation of your permission to use your educational records and to contact you for a follow-up study.

How long will I be in the study?

Your participation in this study will last approximately one hour in length.

What are the possible risks or discomforts?

All research carries risk. The standard for minimal risk is that which is found in everyday life. With the research team's efforts to maintain confidentiality, risk of your identification is unlikely; however, there is a potential of breach of confidentiality with any research. Safeguards are in place to minimize the risk of breach of confidentiality, as outlined in the confidentiality section of this consent form. Risks greater than those encountered in everyday life are not anticipated. Your participation in the research study will have no effect upon your grades or class standing in any course.

Are there any potential benefits?

There are no direct benefits to participating in the study. However, having practice of describing your problem solving process could be useful in job interviews. Your responses may help to improve the development of content, assessment, and pedagogy that help students better understand difficult concepts in Continuous-Time Signals and Systems course ([redacted]).

Will I receive payment or other incentive?

The compensation for the complete participation in this study is twenty dollars. You will receive a check request form that can be redeemed in the [redacted] Business Office for your compensation for completing the study. You will receive the compensation of twenty dollars at the end of the interview for this study irrespective of whether you allow us to access your academic records or not and whether you choose to participate in the follow-up study or not. There are no extra costs to participants.

Will information about me and my participation be kept confidential?

Confidentiality regarding your participation will be maintained. The faculty at [redacted] will not be given any knowledge of your participation in this study, and all reasonable efforts will be made to ensure that your data remains anonymous to them. Any notes associated with this interview will be used without reference to your name. If you agree to participate in a follow-up study, we will save your contact information separately in a password protected word document indefinitely; otherwise any information that can connect the data with you will be destroyed after the study is complete. The data collected in this interview may be used for future research purposes even if you choose not to participate in the follow-up study, but it will be de-identified and your identity will not be traceable from the data by anyone including any researcher in this study. If you choose to participate in the follow-up study, your information will be saved in a password protected word document that will be saved on a password protected portable hard-drive which will be kept locked in the PI's office at Purdue University indefinitely. Only the PI and Farrah Fayyaz will have access to your information after the completion of the study. Transcriptions will be made from the interview audio files, with all identifying information removed. Video recording of the screen of the Microsoft Surface Pro will not be transcribed. All data will be stored on a password protected portable hard drive which will be kept in a locked filing cabinet in the PI's office at Purdue University. Your consent forms will also be kept locked in the PI's office at Purdue University and retained indefinitely. Only Farrah Fayyaz will know the actual identity of the participants. Only the unidentifiable data will be available to the personnels within the research team of this study. No one outside this study will have access to even the de-identified data. The results of this study will be presented in a way that any information about the participants will be unidentifiable. The project's research records may be reviewed by NSF (as the agency that is funding this research project) and by departments at Purdue University and [redacted] Technology responsible for regulatory and research oversight.

What are my rights if I take part in this study?

You do not have to participate in this research project. Your participation in this study is voluntary. You may choose not to participate or, if you agree to participate, you can withdraw your participation at any time without penalty or loss of benefits to which you are otherwise entitled. However, as described above, data will be de-identified such that the participants' names and other critical identification information will be removed, which means once the identity is removed from the data we will no longer be able to identify your data from any of the other data. This restricts your choice to withdraw from participating in the study only up to the time when the association of your name with the data is destroyed. Your participation will not affect any of your grades.

Who can I contact if I have questions about the study?

Farrah Fayyaz will be conducting the procedures of this study. For any questions related to that, she will be your first point of contact. You can contact her at ffayyaz@purdue.edu or [redacted]

This work is being carried out in cooperation between Dr. Ruth Streveler in School of Engineering Education at Purdue University and Dr. [redacted] in the Department of Electrical and Computer Engineering at [redacted] If you have any questions about this research project, you can contact Dr. Streveler at rastreve@purdue.edu or [redacted] or Dr. [redacted] at [redacted] or [redacted].

If you have questions about your rights while taking part in the study or have concerns about the treatment of research participants, please contact [redacted] at Institutional Review Board (IRB) at [redacted] at [redacted] or [redacted] or at the Human Research Protection Program at Purdue University at (765) 494-5942, email (irb@purdue.edu) or write to:

Human Research Protection Program - Purdue University
Ernest C. Young Hall, Room 1032

155 S. Grant St.,
West Lafayette, IN 47907-2114

Documentation of Informed Consent

I have had the opportunity to read this consent form and have the research study explained. I have had the opportunity to ask questions about the research study, and my questions have been answered. I am prepared to participate in the research study described above. I will be offered a copy of this consent form after I sign it.

☐ I give consent for the researchers to contact me again for follow-up study.____________________
(please initial)

☐ I give consent for the researchers to use my education records.____________________
(please initial)

Participant's Signature ____________________ Date ____________________

Participant's Name ____________________

Researcher's Signature ____________________ Date ____________________

Appendix H - Recruitment email

Subject: Would You Like to Participate in an Engineering Education Research Project and Earn $20?

I, Farrah Fayyaz, am working under directions of [redacted] on an NSF-sponsored project to research how students think about concepts in Continuous-Time Signals and Systems (ECE [redacted]) and we need your help. Here are the specifics:

WHO?
We are looking for **undergraduate electrical and computer engineering students who have passed Continuous-Time Signals and Systems (ECE [redacted])**

WHAT?
Get paid to participate in an interview about engineering concepts. Your responses are confidential! Your participation is ***strictly voluntary***. Participation will consist of participating in a **1-hour** one-on-one **interview**. Your **participation will not affect your grades** in any of your classes

WHAT'S THE PAYMENT?
Participants will receive $20 for their time at the end of their interview.

WHEN?
Interviews will be scheduled according to participants' availability (including weekends), starting **March 10, 2014 till March 16, 2014**.

PLACE?
[redacted]

HOW DO I SIGN UP?
Please email Farrah Fayyaz (ffayyaz@purdue.edu) to participate and include:

- Name, email and phone number
- Your willingness to give us permission to access your academic records (grades in physics, engineering, and mathematics courses; score in Signals and Systems concept inventory; Learning Style Index scores) so we can better understand your preparation for the ECE[redacted] content. Please note that you can participate in the study even if you choose to not allow us to access your academic records.
- Preferred days/times for your availability

FOR ANY FURTHER QUESTIONS/CONCERNS
If you have any questions or concerns about the study please email Dr [redacted] or Farrah Fayyaz ([redacted]

Appendix I - Announcement script for recruitment in classes at Iris University

Hi. I am Farrah Fayyaz, a PhD student at Purdue in School of Engineering Education. We are trying to identify why concepts taught in Continuous time Signals and Systems courses are difficult to learn. Our goal is that our study will help to make this course easier to understand for students. We will be conducting one-on-one interviews with students and lean about their thought processes while solving questions related to Continuous Time Signals and Systems. We are looking for participants to help us with our study. All of you are eligible to be interviewed if you have already passed this course.

Your participation in this study will be completely voluntary. Your decision to participate will not affect your grades in any course. You will not be judged based on right or wrong answers. We are only interested to look at your thought processes. Your interview will last for an hour and you will receive $20 for participating. We hope this experience might give you insights that will help you in your learning too.

I am sending around two handouts. The first page has my contact information and more details about the study. On the second page, I will like you to write down your name and email address if you will like to volunteer. You can give them to me on your way out.

Please review the sheets that I have handed out. If you would like to receive more information about this project, or if you decide to participate later, please send me an email.

Thank you so much for your time.

Appendix J - Iris University's Continuous Time Signals and Systems course outline

Electrical and Computer Engineering Continuous-Time Signals and Systems

Catalog Description
Continuous-Time Signals Systems 3R-3L-4C F, W, S, Signal modeling. Fourier series and Fourier transforms. Response of systems to periodic and aperiodic signals. Filter characterization and design. Ideal and practical sampling. Use of numerical analysis software. Integral laboratory

Required/Elective
Required

Class/Laboratory Schedule
Three 50-minute lectures and one 150-minute lab per week.

Prerequisites
Dynamic Systems and Differential Equations I and II

Textbook(s)
Class Notes.

Course Objectives
After successful completion of this course, students will be able to:

1. Demonstrate an ability to represent a variety of signals and system responses both mathematically and graphically.
2. Demonstrate an ability to appropriately characterize signals (i.e. power vs. energy, periodic vs. aperiodic, …).
3. Demonstrate an ability to determine the average power, DC value, and RMS value of a signal.
4. Demonstrate the ability to represent a periodic signal by a Fourier series, and describe its frequency content from that representation.
5. Demonstrate the ability to predict the output of a filter excited by an arbitrary periodic or aperiodic input waveform.
6. Demonstrate the ability to represent a signal or an impulse response by a Fourier transform.
7. Show proficiency in using standard Fourier transform pairs and properties to simplify calculation of forward and inverse transforms of both energy and power signals.

8. Be able to classify filters as lowpass, highpass, bandpass, or bandstop. Interpret lowpass and bandpass filter specifications, and understand the concept of distortion.
9. Find and sketch the time and frequency domain representation of a signal after sampling.
10. Perform time and frequency domain measurements in the laboratory and be able to describe the relationship between them

Course Topics

1. Complex numbers and complex exponentials
2. Periodicity and the fundamental period for summed periodic signals
3. Finding and understanding the Fourier Series of periodic signals
4. Finding and understanding the Fourier Transform for various aperiodic signals
5. Using the Fourier Series and Transforms to determine the output of a system for any arbitrary periodic or aperiodic input
6. Characteristics of ideal and real filters.
7. Transfer functions of real filters in the Laplace and Fourier Domain.
8. Relationship of pole-zero plots to a filter's transfer function in the frequency domain.
9. Basic filter types and functions (Butterworth, Chebychev, and Bessel)(highpass, bandpass, lowpass).
10. Using circuit analysis techniques to derive the transfer function of a filter.
11. Op-amp implementations of 1^{st} and 2^{nd} order circuits and how to cascade these circuits to produce higher order filters.
12. Impulse sampling, pulse sampling, and ZOH sampling.
13. Nyquist criteria.
14. Recovering a sampled signal, including the impulse response of the recovery filter.
15. Aliasing and anti-aliasing filters.

Lab Topics

- Linearity and time-invariance in the time and frequency domains
    - Measuring the power spectrum of periodic and aperiodic signals
    - Signal-to-Noise ratio
- Modulation
- Sampling
- Filter characteristics and filtering signals
- Relating time and frequency domain representations and measurements of signals

ABET Criteria

Objectives/ Outcomes	1	2	3	4	5	6	7	8	9	10
a	•	•	•	•	•	•	•	•	•	•
b										•
c										
d										
e										
f										
g										•
h										
i										
j										
k										•

Professional Component

This course provides 4 credits of engineering topics, 0 credits of basic science, 0 credits of basic math, and 0 credit of engineering design.

Appendix K - Codebook for this study

K.1. Problematic Reasonings

Definition: Reasoning is a purposeful effort to generate justifiable conclusions and make sense of the problem (Definition specifically created for this study). Problematic reasoning is a reasoning that has the potential to hinder conceptual understanding and cultivate misconceptions (Definition specifically created for this study).

K.1.1. Problematic Reasonings Related to Content Area of Signal Representations and Operations

SRO4. Any property of a signal is limited within the duration of the signal itself.

Definition: Students think that any property of a signal is limited within the duration of the signal

Example:

The participants were asked to find and draw the integral $s_1(t)$ of a rectangular function $v_1(t)$ shown in Figure below

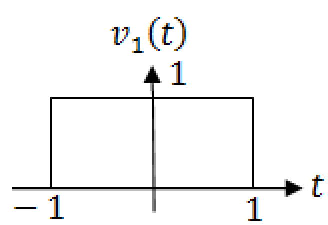


Figure K.1. Rectangular function from the protocol question given to the participants during the interview

For this question, Tom's answer and graph is shown below:

Let's see. Area beneath the curve from negative 1 to 1. ...so, at first glance, I would say it's constant... Hmm. So, my thoughts on this would be, if I were going to take the integral of this function from negative 1 to 1, it's just 1 from those during that time, then I would end up with maybe 1 to 1 of the $v_1(t)$, which is just 1 dt. I'm going to get t that goes from evaluated at negative 1 to 1. And so, if I take this and say 1 minus negative 1, I'm going to get 2. And... I guess it would just be that then, negative 1 to 1, 2, because it's constant. This was constant 1 during that range, and if I integrate that, I'll end up getting a larger value. That's how I would-- that's what I would say. (Tom)

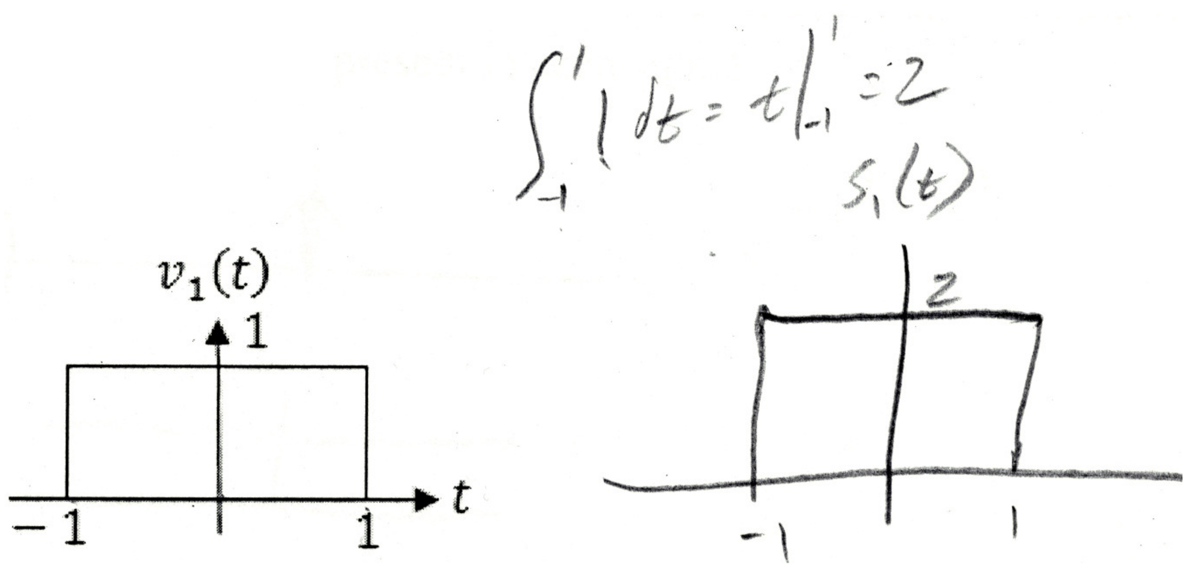


Figure K.2. Tom's response for area under the rectangular function

SRO5. δ(t) or δ(ω) are functions like x(t) which varies according to whatever value t takes on.

Definition: The participants thought that the placement of an impulse function δ(t) on the time axis varies according to whatever value t takes on just like in any function x(t).

Example: While explaining the spectrum of a constant one, Carl said

If we're doing one, which becomes two pi Dirac of omega, I guess then that all frequencies would be present, but a Dirac only takes place at one frequency, because I believe a Dirac function has a-- it's defined as having like a width of zero over an infinite height, so it has an area of one, but since this has two pi in front of it it would just be a Dirac with an area of two pi at one specific frequency, and I think that would depend on what time we have. But, I mean, it'll be zero for most frequencies except for omega. (Carl)

SRO6. The product of any function and an impulse function is a constant.

Definition: The participants thought that the product of any function and an impulse function is a constant function

Example: When asked to plot and explain the Fourier transform of $z(t) = t^2\delta(t-1)$ Luke said,

Okay, so in this case, for z(t), because that's an impulse response we know that it only happens at that point in time. So this has a time delay and so we have to wait until one, and then when we get to one that's when we turn off-- or when we turn on and it goes immediately off. So that means we're looking at zero, one, and then I'm just gonna kind of ghost out what the picture looks like. We know that we have a magnitude of one. Well, it happens at one because z-- oops-- z(1) equals one squared times one. So we just get one. But we're only doing the Fourier transform of that particular point... the Fourier transform of one should be one. (Luke)

K.1.2. Problematic Reasonings Related to Content Area of Frequency Analysis

FA1. A periodic signal in the time domain is also periodic in the frequency domain.

Definition: The data showed that the participants thought that a periodic signal in the time domain is also periodic in the frequency domain.

Example: The participants were given an aperiodic rectangular function and a periodic rectangular function and were asked to explain if the knowledge of the frequencies present in a periodic signal help to determine the frequencies present in a corresponding aperiodic signal is. Carl said,

> Generally I think if something's periodic in the time domain it's probably also periodic in the frequency domain. Like if we have-- this is an impulse train I believe, right?... Then the impulse train in the frequency domain is also an impulse train with a different time shift and a different area, so knowing that this one becomes periodic and knowing that to get this you just convolved this with a Dirac function so it gives you another periodic single, then that would make me think that this would also be periodic... I'm drawing in dots because I'm not entirely positive where that next one is. I just know what it's supposed to look like. And then if that was at eight pi then you would have another one at negative eight pi as well, because it's going to repeat periodically. And because it's a sinc function there actually will be a lot of aliasing, and so you just eventually have a train of sync functions, but knowing what frequencies are present in this first sync can help you determine the spacing for the rest of them. (Carl)

FA2. Signal representation in the time domain is same representation in the frequency domain.

Definition: The data showed that the participants thought that the signal representation in the time domain is the same as the signal representation in the frequency domain.

Example: In a question Luke while trying to find Fourier transform of $z(t) = t^2\delta(t-1)$ said that $z(1) = 1$ and so "Fourier transform of one should be one."

FA3. A constant in the frequency domain means no frequency as it has no ω in it.

Definition: The data showed that the participants employed problematic reasonings in explaining what frequencies were represented by a constant in the frequency domain and thought no ω means no frequency.

Example: Caleb while telling about out the frequency components of an impulse function said, So, $v_2(t)$ is equal to δ(t), which is an impulse in time domain, and for the Fourier in the frequency domain, it's just 1. So, I don't think there is a frequency represented in this signal... Because there's just no omega in the frequency domain, just a 1. (Caleb)

FA4. Phase shift means shifting the phase plot of a signal in the frequency domain

Definition: The data shows that the participants interchanged the concept of time shift in the time domain with the concept of phase shift in the frequency domain.

Example: The participants were given the signal $h(t) = \sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$ and were asked to explain the concept of the time shift and the phase shift using $h(t)$ as an example. Jim said, "And then for a phase shift... in the frequency domain, it would look just like the time shift does in the time domain."

K.1.3. Problematic Reasonings Related to Content Area of System Analysis

SA1. Convolution and multiplication are interchangeable.

Definition: The data showed that the participants interchanged the concept of convolution with multiplication.

Example: The participants were given two same signals expressed separately in the frequency domain and in the time domain and were asked to explain if the resultant signals from convolution of these signals separately in the time domain ($h(t) * x(t)$) and in the frequency domain ($h(f) * x(f)$) will be the same or not. Matt said, "Yeah, I think it will change... Because the Fourier transform is changed from time domain to the frequency domain. I think h(t) will give the range of the y(t)... So it's from -1 to 1... y(t) would be from -1 to 1 too... Because there's a filter so it will filter out the parts that doesn't include in that. "

SA2. Concept of time-invariance of a system is interchangeable with the literal meaning of time invariance.

Definition: The data showed that the participants thought that the concept of the time-invariance of a system could be explained with the literal meaning of time-invariance.

Example: The participants were asked to explain whether given systems were time-invariant or not. John said, "Time-invariant, it would be the same shape output but not necessarily the same time. And not time-invariant would be the output would be the same at the same time."

K.2. Mistakes

The incorrect responses of the participants for which there is not enough evidence for reasonings behind them are called mistakes (Definition specifically created for this study).

Mistake1. Engaging with the powers of exponential functions

Definition: The data showed that the participants made mistakes in dealing with the powers of exponentials.

Example: Following is the example when Bill tried to find Fourier transform of t-square.

$$\int_{-\infty}^{\infty} t^{2} e^{-jwt}$$

$$e^{-jw} \int_{-\infty}^{\infty} t^{2} e^{t}$$

Figure K.3. Bill's attempt to find Fourier transform of t-square

Mistake2. Translating a mathematical equation

Definition: The data showed that the participants made mistakes in translating a mathematical equation, for example, to a graph.

Example: Lily drew the graph of t-square and mistakenly assumed it will be zero before time = 0 as shown below.

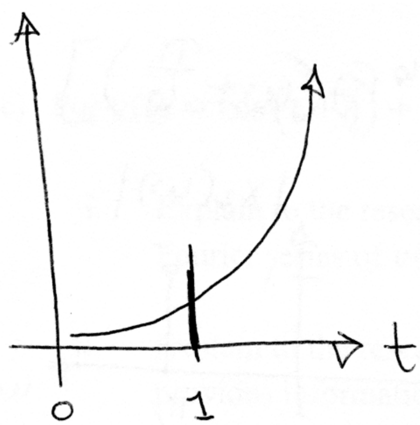


Figure K.4. Lily's attempt to draw t-square

Mistake3. Engaging with a unit step function

Definition: The data showed that the participants made mistakes when engaging with a unit step function.

Example: Lily when trying to find Fourier transform of $x(t) = t^2u(t)u(1-t)$, replaced limits of integral from 0 to 1 but did not remove *u(t)*s from the expression shown in the figure below. Because of unit step functions still in the integral she could not solve the integral.

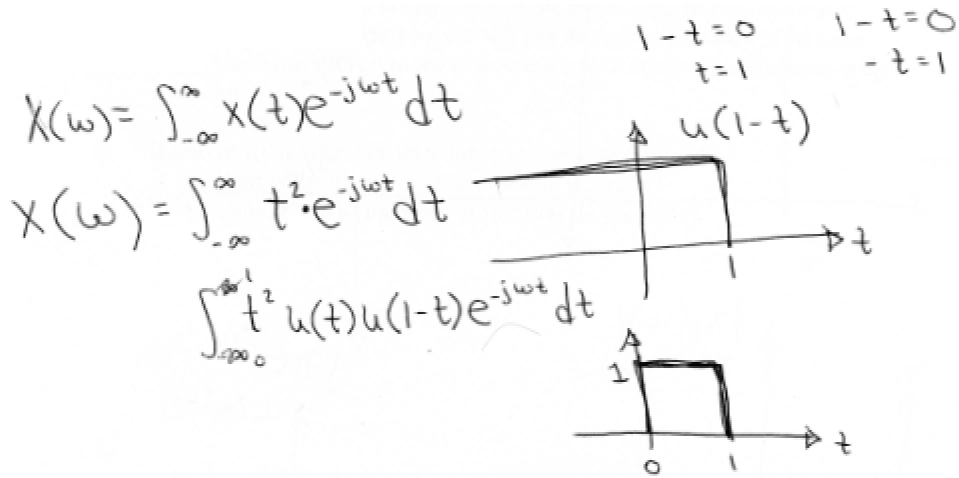


Figure K.5. Lily's working of using unit step functions inside an integral

Mistake4. Engaging with an impulse function

Definition: The data showed that the participants made mistakes when engaging with an impulse function.

Example: The participants were asked to explain what will be the Fourier transform of $d(t) = t^2$ and then Fourier transform of $z(t) = t^2\delta(t-1)$. Jim said, "That that would be the same math here, then plot out. Except the integral would go from one to infinity."

Mistake5. Performing time shift and time scale operations combined

Definition: The data showed that the participants made mistakes when performing time shift and time scale operations combined.

Example: Following is an example of Megan's attempt to draw $sin\,(\frac{\pi}{2}t - \frac{\pi}{4})$:

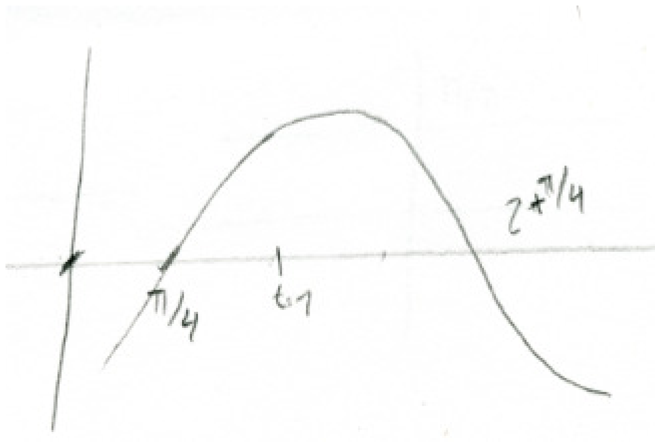


Figure K.6. Plot of $sin\ (\frac{\pi}{2}t - \frac{\pi}{4})$ drawn by Megan

Mistake6. Interchange similar terms and concepts

Definition: The data showed that the participants made mistakes of interchanging similar terms and concepts

Example: Matt interchanged the formula for Fourier series of a signal when performing a similar operation on a periodic signal and mistakenly used the concepts of Fourier series analysis when dealing with periodic signal. He said, "Oh since this one's a periodic, the graph of $s_1(t)$ is a straight line. So this one's supposed to be a straight line too because it's kind of like say a series graph so we only focus on one period of it. So I think we don't actually need to care about the others. We just need to probably make a comment and say this one is a series."

K.3. Missing Conceptual Knowledge

Missing conceptual knowledge is the lack of conceptual knowledge exhibited by the participants in their problem solving approaches during interviews (definition specifically created for this study). This lack of knowledge was evidenced from 1) the participants' confrontation about the lack of their knowledge when they were prompted to explain their responses, or 2) the responses that showed failure to appeal to some useful knowledge to correctly answer the given question.

4. Difference in the use of graphical representations of discrete and continuous impulse functions

Definition: The data shows that the participants lacked the knowledge of the difference in the use of the graphical representations of the discrete and continuous impulse functions.

Example: During the interview, Jake drew an impulse and I asked him to explain why he chose to use that particular representation of the impulse function. He said, "I don't know. I sometimes use this, sometimes use this one."

5. Conceptual understanding of Fourier Analysis

Definition: The responses of the students revealed that the participants leaned towards solving questions related to Fourier analysis using either commonly used Fourier transform pairs given in Fourier transform table or through integration and displayed lack of acumen in Fourier analysis if they were encountered with question, which was not easy to answer through any of these two preferred methods.

Example 1: Ability to recognize that a function originally expressed in the form of sinusoids or exponentials is already expressed in the form of Fourier series or transform.

The participants were asked to find Fourier series and transform of $v(t) = \cos\left(t + \frac{\pi}{4}\right) + 3\sin(7t)$. Luke said, "I can honestly not think of how to do the Fourier series, and so is it okay if I skip and go on?"

Example 2: The data showed that the participants demonstrated a lack of knowledge of the units of Fourier series and Fourier transform.

The participants were asked to discuss if the units of a signal v(t) is volts, what will be the units of Fourier series and Fourier transform of that signal. Kevin said, "I think it's just a magnitude. So units of the magnitude as in unit less?"

Example 3: The data showed that the participants demonstrated a lack of ability to identify that Fourier transform of a signal may not exist

During the interviews, the participants were asked to explain how they will find and plot Fourier transform of $d(t) = t^2$. Luke solved the whole integration and found infinity in the answer and said, " But I don't want to say that omega is just zero, but if it was zero then that makes this little term one and so then we're looking at t-squared throughout infinite time, which kind of blows up. So I feel like there's another way to really-- to look at it better."

Example 4: Ability to identify that Fourier series of an aperiodic signal does not exist

The data showed that the participants demonstrated a lack of ability to identify that Fourier series of an aperiodic signal does not exist. A aperiodic signal was shown to the participants and they were asked to explain what the Fourier series of that signal will be. Luke said, "I'm thinking of like you can take your data, like certain points from that, and build up a more and more and more exact approach to it. And so, this, the Fourier transform, tries to find not, you know, the precise to the, you know, millionth decimal, but like the good, general statement of, you know, this is what, excuse me, this is what the like first initial terms and the Fourier transform-- or the Fourier series are, sum up to."

6. Ability to translate a function from one representation to another

Definition: The data showed that the participants demonstrated a lack of ability to translate a function from one representation to another.

Example: Ryan while explaining a problem said, "The mathematical explanation is sufficient for me to understand it so I never really looked into it more, I guess."

VITA

VITA

Farrah Fayyaz received her B.S. and M.S. degrees in Electrical Engineering at the University of Engineering and Technology, Lahore in 2000 and 2009 respectively. She taught undergraduate Electrical Engineering courses in Pakistan for almost nine years before starting her Ph.D. in Engineering Education at Purdue University in 2011. She will be returning to Pakistan in December 2014 as a faculty member, following the completion of her degree.

She received a Fulbright Fellowship for her PhD (2011-2015) and a Ross Fellowship from Purdue for the academic year 2011-2012. Her future research plans include cross-cultural studies on conceptual understanding of undergraduate electrical engineering students.